Question ID 5822c232

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: 5822c232

Which expression is equivalent to $\frac{y+12}{x-8}+\frac{y(x-8)}{x^2y-8xy}$?

A. $\frac{xy+y+4}{x^3y-16x^2y+64xy}$

B. $\frac{xy+9y+12}{x^2y-8xy+x-8}$

C. $\frac{xy^2+13xy-8y}{x^2y-8xy}$

D. $\frac{xy^2+13xy-8y}{x^3y-16x^2y+64xy}$

ID: 5822c232 Answer

Correct Answer: C

Rationale

Choice C is correct. Factoring the denominator in the second term of the given expression gives $\frac{y+12}{x-8}+\frac{y(x-8)}{xy(x-8)}$. This expression can be rewritten with common denominators by multiplying the first term by $\frac{xy}{xy}$, giving $\frac{xy(y+12)}{xy(x-8)}+\frac{y(x-8)}{xy(x-8)}$. Adding these two terms yields $\frac{xy(y+12)+y(x-8)}{xy(x-8)}$. Using the distributive property to rewrite this expression gives $\frac{xy^2+12xy+xy-8y}{x^2y-8xy}$. Combining the like terms in the numerator of this expression gives $\frac{xy^2+13xy-8y}{x^2y-8xy}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 4443355f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: 4443355f

The expression $4x^2 + bx - 45$, where b is a constant, can be rewritten as $(hx + k)(x + j)$, where h, k, and j are integer constants. Which of the following must be an integer?

A. $\frac{b}{h}$

B. $\frac{b}{k}$

C. $\frac{45}{h}$

D. $\frac{45}{k}$

ID: 4443355f Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that $4x^2 + bx - 45$ can be rewritten as $(hx + k)(x + j)$. The expression $(hx + k)(x + j)$ can be rewritten as $hx^2 + jhx + kx + kj$, or $hx^2 + (jh + k)x + kj$. Therefore, $hx^2 + (jh + k)x + kj$ is equivalent to $4x^2 + bx - 45$. It follows that $kj = -45$. Dividing each side of this equation by k yields $j = \frac{-45}{k}$. Since j is an integer, $-\frac{45}{k}$ must be an integer. Therefore, $\frac{45}{k}$ must also be an integer.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID a1397504

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: a1397504

$$0.36x^2 + 0.63x + 1.17$$

The given expression can be rewritten as $a(4x^2 + 7x + 13)$, where a is a constant. What is the value of a?

ID: a1397504 Answer

Correct Answer: .09, 9/100

Rationale

The correct answer is $.09$. It's given that the expression $0.36x^2 + 0.63x + 1.17$ can be rewritten as $a(4x^2 + 7x + 13)$. Applying the distributive property to the expression $a(4x^2 + 7x + 13)$ yields $4ax^2 + 7ax + 13a$. Therefore, $0.36x^2 + 0.63x + 1.17$ can be rewritten as $4ax^2 + 7ax + 13a$. It follows that in the expressions $0.36x^2 + 0.63x + 1.17$ and $4ax^2 + 7ax + 13a$, the coefficients of x^2 are equivalent, the coefficients of x are equivalent, and the constant terms are equivalent. Therefore, $0.36 = 4a$, $0.63 = 7a$, and $1.17 = 13a$. Solving any of these equations for a yields the value of a. Dividing both sides of the equation $0.36 = 4a$ by 4 yields $0.09 = a$. Therefore, the value of a is 0.09. Note that .09 and 9/100 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID eafd61d3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: eafd61d3

The expression $(3x - 23)(19x + 6)$ is equivalent to the expression $ax^2 + bx + c$, where a, b, and c are constants. What is the value of b?

ID: eafd61d3 Answer

Correct Answer: -419

Rationale

The correct answer is -419. It's given that the expression $(3x - 23)(19x + 6)$ is equivalent to the expression $ax^2 + bx + c$, where a, b, and c are constants. Applying the distributive property to the given expression, $(3x - 23)(19x + 6)$, yields $(3x)(19x) + (3x)(6) - (23)(19x) - (23)(6)$, which can be rewritten as $57x^2 + 18x - 437x - 138$. Combining like terms yields $57x^2 - 419x - 138$. Since this expression is equivalent to $ax^2 + bx + c$, it follows that the value of b is -419.

Question Difficulty: Hard

Question ID 68fb4847

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: 68fb4847

Which expression is equivalent to $\frac{42a}{k} + 42ak$, where $k > 0$?

A. $\frac{84a}{k}$

B. $\frac{84ak^2}{k}$

C. $\frac{42a(k+1)}{k}$

D. $\frac{42a(k^2+1)}{k}$

ID: 68fb4847 Answer

Correct Answer: D

Rationale

Choice D is correct. Two fractions can be added together when they have a common denominator. Since $k > 0$, multiplying the second term in the given expression by $\frac{k}{k}$ yields $\frac{(42ak)k}{k}$, which is equivalent to $\frac{42ak^2}{k}$. Therefore, the expression $\frac{42a}{k} + 42ak$ can be written as $\frac{42a}{k} + \frac{42ak^2}{k}$ which is equivalent to $\frac{42a+42ak^2}{k}$. Since each term in the numerator of this expression has a factor of $42a$, the expression $\frac{42a+42ak^2}{k}$ can be rewritten as $\frac{42a(1)+42a(k^2)}{k}$, or $\frac{42a(1+k^2)}{k}$, which is equivalent to $\frac{42a(k^2+1)}{k}$.

Choice A is incorrect. This expression is equivalent to $\frac{42a}{k} + \frac{42a}{k}$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This expression is equivalent to $\frac{42a}{k} + 42a$.

Question Difficulty: Hard

Question ID ec3981ea

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: ec3981ea

If $k - x$ is a factor of the expression $-x^2 + \frac{1}{29}nk^2$, where n and k are constants and $k > 0$, what is the value of n?

A. -29

B. $-\frac{1}{29}$

C. $\frac{1}{29}$

D. 29

ID: ec3981ea Answer

Correct Answer: D

Rationale

Choice D is correct. If $k - x$ is a factor of the expression $-x^2 + \left(\frac{1}{29}\right)nk^2$, then the expression can be written as $(k - x)(ax + b)$, where a and b are constants. This expression can be rewritten as $akx + bk - ax^2 - bx$, or $-ax^2 + (ak - b)x + bk$. Since this expression is equivalent to $-x^2 + \left(\frac{1}{29}\right)nk^2$, it follows that $-a = -1$, $ak - b = 0$, and $bk = \left(\frac{1}{29}\right)nk^2$. Dividing each side of the equation $-a = -1$ by -1 yields $a = 1$. Substituting 1 for a in the equation $ak - b = 0$ yields $k - b = 0$. Adding b to each side of this equation yields $k = b$. Substituting k for b in the equation $bk = \left(\frac{1}{29}\right)nk^2$ yields $k^2 = \left(\frac{1}{29}\right)nk^2$. Since k is positive, dividing each side of this equation by k^2 yields $1 = \left(\frac{1}{29}\right)n$. Multiplying each side of this equation by 29 yields $29 = n$.

Alternate approach: The expression $x^2 - y^2$ can be written as $(x - y)(x + y)$, which is a difference of two squares. It follows that $\left(\frac{1}{29}\right)nk^2 - x^2$ is equivalent to $\left(\left(\sqrt{\frac{1}{29}n}\right)k - x\right)\left(\left(\sqrt{\frac{1}{29}n}\right)k + x\right)$. It's given that $k - x$ is a factor of $-x^2 + \left(\frac{1}{29}\right)nk^2$, so the factor $\left(\sqrt{\frac{1}{29}n}\right)k - x$ is equal to $k - x$. Adding x to both sides of the equation $\left(\sqrt{\frac{1}{29}n}\right)k - x = k - x$ yields $\left(\sqrt{\frac{1}{29}n}\right)k = k$. Since k is positive, dividing both sides of this equation by k yields $\sqrt{\frac{1}{29}n} = 1$. Squaring both sides of this equation yields $\frac{1}{29}n = 1$. Multiplying both sides of this equation by 29 yields $n = 29$.

Choice A is incorrect. This value of n gives the expression $-x^2 + \left(\frac{1}{29}\right)(-29)k^2$, or $-x^2 - k^2$. This expression doesn't have $k - x$ as a factor.

Choice B is incorrect. This value of n gives the expression $-x^2 + \left(\frac{1}{29}\right)\left(-\frac{1}{29}\right)k^2$, or $-x^2 + \left(-\frac{1}{841}\right)k^2$. This expression doesn't have $k - x$ as a factor.

Choice C is incorrect. This value of n gives the expression $-x^2 + \left(\frac{1}{29}\right)\left(\frac{1}{29}\right)k^2$, or $-x^2 + \left(\frac{1}{841}\right)k^2$. This expression doesn't have $k - x$ as a factor.

Question Difficulty: Hard

Question ID 9d146dca

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: 9d146dca

Which of the following expressions has a factor of $x + 2b$, where b is a positive integer constant?

A. $3x^2 + 7x + 14b$

B. $3x^2 + 28x + 14b$

C. $3x^2 + 42x + 14b$

D. $3x^2 + 49x + 14b$

ID: 9d146dca Answer

Correct Answer: D

Rationale

Choice D is correct. Since each choice has a term of $3x^2$, which can be written as $(3x)(x)$, and each choice has a term of $14b$, which can be written as $(7)(2b)$, the expression that has a factor of $x + 2b$, where b is a positive integer constant, can be represented as $(3x + 7)(x + 2b)$. Using the distributive property of multiplication, this expression is equivalent to $3x(x + 2b) + 7(x + 2b)$, or $3x^2 + 6xb + 7x + 14b$. Combining the *x*-terms in this expression yields $3x^2 + (7 + 6b)x + 14b$. It follows that the coefficient of the *x*-term is equal to $7 + 6b$. Thus, from the given choices, $7 + 6b$ must be equal to 7, 28, 42, or 49. Therefore, $6b$ must be equal to 0, 21, 35, or 42, respectively, and b must be equal to $\frac{0}{6}$, $\frac{21}{6}$, $\frac{35}{6}$, or $\frac{42}{6}$, respectively. Of these four values of b, only $\frac{42}{6}$, or 7, is a positive integer. It follows that $7 + 6b$ must be equal to 49 because this is the only choice for which the value of b is a positive integer constant. Therefore, the expression that has a factor of $x + 2b$ is $3x^2 + 49x + 14b$.

Choice A is incorrect. If this expression has a factor of $x + 2b$, then the value of b is 0, which isn't positive.

Choice B is incorrect. If this expression has a factor of $x + 2b$, then the value of b is $\frac{21}{6}$, which isn't an integer.

Choice C is incorrect. If this expression has a factor of $x + 2b$, then the value of b is $\frac{35}{6}$, which isn't an integer.

Question Difficulty: Hard

Question ID 05cec180

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: 05cec180

Which expression is equivalent to $\frac{4}{4x-5} - \frac{1}{x+1}$?

A. $\frac{1}{(x+1)(4x-5)}$

B. $\frac{3}{3x-6}$

C. $-\frac{1}{(x+1)(4x-5)}$

D. $\frac{9}{(x+1)(4x-5)}$

ID: 05cec180 Answer

Correct Answer: D

Rationale

Choice D is correct. The expression $\frac{4}{4x-5} - \frac{1}{x+1}$ can be rewritten as $\frac{4}{4x-5} + \frac{(-1)}{x+1}$. To add the two terms of this expression, the terms can be rewritten with a common denominator. Since $\frac{x+1}{x+1} = 1$, the expression $\frac{4}{4x-5}$ can be rewritten as $\frac{(x+1)(4)}{(x+1)(4x-5)}$. Since $\frac{4x-5}{4x-5} = 1$, the expression $\frac{-1}{x+1}$ can be rewritten as $\frac{(4x-5)(-1)}{(4x-5)(x+1)}$. Therefore, the expression $\frac{4}{4x-5} + \frac{(-1)}{x+1}$ can be rewritten as $\frac{(x+1)(4)}{(x+1)(4x-5)} + \frac{(4x-5)(-1)}{(4x-5)(x+1)}$, which is equivalent to $\frac{(x+1)(4)+(4x-5)(-1)}{(x+1)(4x-5)}$. Applying the distributive property to each term of the numerator yields $\frac{(4x+4)+(-4x+5)}{(x+1)(4x-5)}$, or $\frac{(4x+(-4x))+(4+5)}{(x+1)(4x-5)}$. Adding like terms in the numerator yields $\frac{9}{(x+1)(4x-5)}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID fead0fc7

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: fead0fc7

The expression $6\sqrt[5]{3^5x^{45}} \cdot \sqrt[8]{2^8x}$ is equivalent to ax^b, where a and b are positive constants and $x > 1$. What is the value of $a + b$?

ID: fead0fc7 Answer

Correct Answer: 361/8, 45.12, 45.13

Rationale

The correct answer is $\frac{361}{8}$. The rational exponent property is $\sqrt[n]{y^m} = y^{\frac{m}{n}}$, where $y > 0$, m and n are integers, and $n > 0$. This property can be applied to rewrite the given expression $6\sqrt[5]{3^5x^{45}} \cdot \sqrt[8]{2^8x}$ as $6\left(3^{\frac{5}{5}}\right)\left(x^{\frac{45}{5}}\right)\left(2^{\frac{8}{8}}\right)\left(x^{\frac{1}{8}}\right)$, or $6(3)(x^9)(2)\left(x^{\frac{1}{8}}\right)$. This expression can be rewritten by multiplying the constants, which gives $36(x^9)\left(x^{\frac{1}{8}}\right)$. The multiplication exponent property is $y^n \cdot y^m = y^{n+m}$, where $y > 0$. This property can be applied to rewrite the expression $36(x^9)\left(x^{\frac{1}{8}}\right)$ as $36x^{9+\frac{1}{8}}$, or $36x^{\frac{73}{8}}$. Therefore, $6\sqrt[5]{3^5x^{45}} \cdot \sqrt[8]{2^8x} = 36x^{\frac{73}{8}}$. It's given that $6\sqrt[5]{3^5x^{45}} \cdot \sqrt[8]{2^8x}$ is equivalent to ax^b; therefore, $a = 36$ and $b = \frac{73}{8}$. It follows that $a + b = 36 + \frac{73}{8}$. Finding a common denominator on the right-hand side of this equation gives $a + b = \frac{288}{8} + \frac{73}{8}$, or $a + b = \frac{361}{8}$. Note that 361/8, 45.12, and 45.13 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 3138e379

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: 3138e379

$$\sqrt[5]{70n}\left(\sqrt[6]{70n}\right)^2$$

For what value of x is the given expression equivalent to $(70n)^{30x}$, where $n > 1$?

ID: 3138e379 Answer

Correct Answer: .0177, .0178, 4/225

Rationale

The correct answer is $\frac{4}{225}$. An expression of the form $\sqrt[k]{a}$, where k is an integer greater than 1 and $a \geq 0$, is equivalent to $a^{\frac{1}{k}}$. Therefore, the given expression, where $n > 1$, is equivalent to $(70n)^{\frac{1}{5}}\left((70n)^{\frac{1}{6}}\right)^2$. Applying properties of exponents, this expression can be rewritten as $(70n)^{\frac{1}{5}}(70n)^{\frac{1}{6}\cdot 2}$, or $(70n)^{\frac{1}{5}}(70n)^{\frac{1}{3}}$, which can be rewritten as $(70n)^{\frac{1}{5}+\frac{1}{3}}$, or $(70n)^{\frac{8}{15}}$. It's given that the expression $\sqrt[5]{70n}\left(\sqrt[6]{70n}\right)^2$ is equivalent to $(70n)^{30x}$, where $n > 1$. It follows that $(70n)^{\frac{8}{15}}$ is equivalent to $(70n)^{30x}$. Therefore, $\frac{8}{15} = 30x$. Dividing both sides of this equation by 30 yields $\frac{8}{450} = x$, or $\frac{4}{225} = x$. Thus, the value of x for which the given expression is equivalent to $(70n)^{30x}$, where $n > 1$, is $\frac{4}{225}$. Note that 4/225, .0177, .0178, 0.017, and 0.018 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 6b56736a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: 6b56736a

Which of the following expressions is(are) a factor of $3x^2 + 20x - 63$?

I. $x - 9$

II. $3x - 7$

A. I only

B. II only

C. I and II

D. Neither I nor II

ID: 6b56736a Answer

Correct Answer: B

Rationale

Choice B is correct. The given expression can be factored by first finding two values whose sum is 20 and whose product is $3(-63)$, or -189. Those two values are 27 and -7. It follows that the given expression can be rewritten as $3x^2 + 27x - 7x - 63$. Since the first two terms of this expression have a common factor of $3x$ and the last two terms of this expression have a common factor of -7, this expression can be rewritten as $3x(x + 9) - 7(x + 9)$. Since the two terms of this expression have a common factor of $(x + 9)$, it can be rewritten as $(3x - 7)(x + 9)$. Therefore, expression II, $3x - 7$, is a factor of $3x^2 + 20x - 63$, but expression I, $x - 9$, is not a factor of $3x^2 + 20x - 63$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID ab245384

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: ab245384

If $4^{8c} = \sqrt[3]{4^7}$, what is the value of c?

ID: ab245384 Answer

Correct Answer: .2916, .2917, 7/24

Rationale

The correct answer is $\frac{7}{24}$. An expression of the form $\sqrt[n]{a^m}$, where m and n are integers greater than 1 and $a \geq 0$, is equivalent to $a^{\frac{m}{n}}$. Therefore, the expression on the right-hand side of the given equation, $\sqrt[3]{4^7}$, is equivalent to $4^{\frac{7}{3}}$. Thus, $4^{8c} = 4^{\frac{7}{3}}$. It follows that $8c = \frac{7}{3}$. Dividing both sides of this equation by 8 yields $c = \frac{7}{24}$. Note that 7/24, .2916, .2917, 0.219, and 0.292 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID bcbf0e45

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	Hard

ID: bcbf0e45

One of the factors of $2x^3 + 42x^2 + 208x$ is $x + b$, where b is a positive constant. What is the smallest possible value of b?

ID: bcbf0e45 Answer

Correct Answer: 8

Rationale

The correct answer is 8. Since each term of the given expression, $2x^3 + 42x^2 + 208x$, has a factor of $2x$, the expression can be rewritten as $2x\left(x^2\right) + 2x(21x) + 2x(104)$, or $2x\left(x^2 + 21x + 104\right)$. Since the values 8 and 13 have a sum of 21 and a product of 104, the expression $x^2 + 21x + 104$ can be factored as $(x + 8)(x + 13)$. Therefore, the given expression can be factored as $2x(x + 8)(x + 13)$. It follows that the factors of the given expression are 2, x, $x + 8$, and $x + 13$. Of these factors, only $x + 8$ and $x + 13$ are of the form $x + b$, where b is a positive constant. Therefore, the possible values of b are 8 and 13. Thus, the smallest possible value of b is 8.

Question Difficulty: Hard