

# Question ID 3eb56220

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

**ID: 3eb56220**

In triangle  $XYZ$ , angle  $Y$  is a right angle, the measure of angle  $Z$  is  $33^\circ$ , and the length of  $\overline{YZ}$  is 26 units. If the area, in square units, of triangle  $XYZ$  can be represented by the expression  $k \tan 33^\circ$ , where  $k$  is a constant, what is the value of  $k$ ?

**ID: 3eb56220 Answer**

Correct Answer: 338

Rationale

The correct answer is 338. The tangent of an acute angle in a right triangle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. In triangle  $XYZ$ , it's given that angle  $Y$  is a right angle. Thus,  $\overline{XY}$  is the leg opposite of angle  $Z$  and  $\overline{YZ}$  is the leg adjacent to angle  $Z$ . It follows that  $\tan Z = \frac{XY}{YZ}$ . It's also given that the measure of angle  $Z$  is  $33^\circ$  and the length of  $\overline{YZ}$  is 26 units. Substituting  $33^\circ$  for  $Z$  and 26 for  $YZ$  in the equation  $\tan Z = \frac{XY}{YZ}$  yields  $\tan 33^\circ = \frac{XY}{26}$ . Multiplying each side of this equation by 26 yields  $26 \tan 33^\circ = XY$ . Therefore, the length of  $\overline{XY}$  is  $26 \tan 33^\circ$ . The area of a triangle is half the product of the lengths of its legs. Since the length of  $\overline{YZ}$  is 26 and the length of  $\overline{XY}$  is  $26 \tan 33^\circ$ , it follows that the area of triangle  $XYZ$  is  $\frac{1}{2}(26)(26 \tan 33^\circ)$  square units, or  $338 \tan 33^\circ$  square units. It's given that the area, in square units, of triangle  $XYZ$  can be represented by the expression  $k \tan 33^\circ$ , where  $k$  is a constant. Therefore, 338 is the value of  $k$ .

Question Difficulty: Hard

# Question ID 630ce120

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

**ID: 630ce120**

The length of a rectangle's diagonal is  $3\sqrt{17}$ , and the length of the rectangle's shorter side is 3. What is the length of the rectangle's longer side?

**ID: 630ce120 Answer**

Correct Answer: 12

Rationale

The correct answer is 12. The diagonal of a rectangle forms a right triangle, where the shorter side and the longer side of the rectangle are the legs of the triangle and the diagonal of the rectangle is the hypotenuse of the triangle. It's given that the length of the rectangle's diagonal is  $3\sqrt{17}$  and the length of the rectangle's shorter side is 3. Thus, the length of the hypotenuse of the right triangle formed by the diagonal is  $3\sqrt{17}$  and the length of one of the legs is 3. By the Pythagorean theorem, if a right triangle has a hypotenuse with length  $c$  and legs with lengths  $a$  and  $b$ , then  $a^2 + b^2 = c^2$ . Substituting  $3\sqrt{17}$  for  $c$  and 3 for  $b$  in this equation yields  $a^2 + (3)^2 = (3\sqrt{17})^2$ , or  $a^2 + 9 = 153$ . Subtracting 9 from both sides of this equation yields  $a^2 = 144$ . Taking the square root of both sides of this equation yields  $a = \pm\sqrt{144}$ , or  $a = \pm 12$ . Since  $a$  represents a length, which must be positive, the value of  $a$  is 12. Thus, the length of the rectangle's longer side is 12.

Question Difficulty: Hard

# Question ID 4ce3d4ed

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 4ce3d4ed

A rectangle is inscribed in a circle, such that each vertex of the rectangle lies on the circumference of the circle. The diagonal of the rectangle is twice the length of the shortest side of the rectangle. The area of the rectangle is  $1,089\sqrt{3}$  square units. What is the length, in units, of the diameter of the circle?

ID: 4ce3d4ed Answer

Correct Answer: 66

Rationale

The correct answer is **66**. It's given that each vertex of the rectangle lies on the circumference of the circle. Therefore, the length of the diameter of the circle is equal to the length of the diagonal of the rectangle. The diagonal of a rectangle forms a right triangle with the shortest and longest sides of the rectangle, where the shortest side and the longest side of the rectangle are the legs of the triangle and the diagonal of the rectangle is the hypotenuse of the triangle. Let  $s$  represent the length, in units, of the shortest side of the rectangle. Since it's given that the diagonal is twice the length of the shortest side,  $2s$  represents the length, in units, of the diagonal of the rectangle. By the Pythagorean theorem, if a right triangle has a hypotenuse with length  $c$  and legs with lengths  $a$  and  $b$ , then  $a^2 + b^2 = c^2$ . Substituting  $s$  for  $a$  and  $2s$  for  $c$  in this equation yields  $s^2 + b^2 = (2s)^2$ , or  $s^2 + b^2 = 4s^2$ . Subtracting  $s^2$  from both sides of this equation yields  $b^2 = 3s^2$ . Taking the positive square root of both sides of this equation yields  $b = s\sqrt{3}$ . Therefore, the length, in units, of the rectangle's longest side is  $s\sqrt{3}$ . The area of a rectangle is the product of the length of the shortest side and the length of the longest side. The lengths, in units, of the shortest and longest sides of the rectangle are represented by  $s$  and  $s\sqrt{3}$ , and it's given that the area of the rectangle is  $1,089\sqrt{3}$  square units. It follows that  $1,089\sqrt{3} = s(s\sqrt{3})$ , or  $1,089\sqrt{3} = s^2\sqrt{3}$ . Dividing both sides of this equation by  $\sqrt{3}$  yields  $1,089 = s^2$ . Taking the positive square root of both sides of this equation yields  $33 = s$ . Since the length, in units, of the diagonal is represented by  $2s$ , it follows that the length, in units, of the diagonal is  $2(33)$ , or **66**. Since the length of the diameter of the circle is equal to the length of the diagonal of the rectangle, the length, in units, of the diameter of the circle is **66**.

Question Difficulty: Hard

# Question ID 0d4eba4d

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

**ID: 0d4eba4d**

One leg of a right triangle has a length of **43.2** millimeters. The hypotenuse of the triangle has a length of **196.8** millimeters. What is the length of the other leg of the triangle, in millimeters?

- A. **43.2**
- B. **120**
- C. **192**
- D. **201.5**

**ID: 0d4eba4d Answer**

Correct Answer: C

Rationale

Choice C is correct. The Pythagorean theorem states that for a right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse. It's given that one leg of a right triangle has a length of **43.2** millimeters. It's also given that the hypotenuse of the triangle has a length of **196.8** millimeters. Let  $b$  represent the length of the other leg of the triangle, in millimeters. Therefore, by the Pythagorean theorem,  $43.2^2 + b^2 = 196.8^2$ , or  $1,866.24 + b^2 = 38,730.24$ . Subtracting **1,866.24** from both sides of this equation yields  $b^2 = 36,864$ . Taking the positive square root of both sides of this equation yields  $b = 192$ . Therefore, the length of the other leg of the triangle, in millimeters, is **192**.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

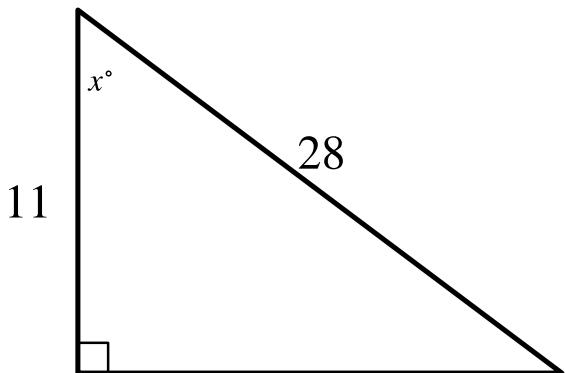
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

# Question ID b9d89675

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: b9d89675



Note: Figure not drawn to scale.

In the triangle shown, what is the value of  $\cos x^\circ$ ?

ID: b9d89675 Answer

Correct Answer: .3928, .3929, 11/28

Rationale

The correct answer is  $\frac{11}{28}$ . The cosine of an acute angle in a right triangle is defined as the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse. In the triangle shown, the length of the leg adjacent to the angle with measure  $x^\circ$  is 11 units and the length of the hypotenuse is 28 units. Therefore, the value of  $\cos x^\circ$  is  $\frac{11}{28}$ . Note that 11/28, .3928, .3929, 0.392, and 0.393 are examples of ways to enter a correct answer.

Question Difficulty: Hard

# Question ID 3224f1ad

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 3224f1ad

The perimeter of an equilateral triangle is 852 centimeters. The three vertices of the triangle lie on a circle. The radius of the circle is  $w\sqrt{3}$  centimeters. What is the value of  $w$ ?

ID: 3224f1ad Answer

Correct Answer: 284/3, 94.66, 94.67

Rationale

The correct answer is  $\frac{284}{3}$ . Since the perimeter of a triangle is the sum of the lengths of its sides, and the given triangle is equilateral, the length of each side is  $\frac{852}{3}$ , or 284, centimeters (cm). Right triangle  $AMO$  can be formed, where  $M$  is the midpoint of one of the triangle's sides,  $A$  is one of this side's endpoints, and  $O$  is the center of the circle. It follows that  $AM$  is  $\frac{284}{2}$ , or 142, cm. Additionally, triangle  $AMO$  has angles measuring  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$ , where the measure of angle  $OMA$  is  $90^\circ$  and the measure of angle  $OAM$  is  $30^\circ$ . It follows that the length of side  $MO$  is half the length of hypotenuse  $AO$ , and the length of side  $AM$  is  $\sqrt{3}$  times the length of side  $MO$ . It's given that  $AO = w\sqrt{3}$  cm.

Therefore,  $MO = \frac{w\sqrt{3}}{2}$  cm and  $AM = \frac{w\sqrt{3}\sqrt{3}}{2}$  cm, which is equivalent to  $AM = \frac{3w}{2}$  cm. Since  $AM = 142$  cm, it follows that  $\frac{3w}{2} = 142$ . Multiplying both sides of this equation by 2 yields  $3w = 284$ . Dividing both sides of this equation by 3 yields  $w = \frac{284}{3}$ . Note that 284/3, 94.66, and 94.67 are examples of ways to enter a correct answer.

Question Difficulty: Hard

# Question ID dec3599b

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

## ID: dec3599b

For two acute angles,  $\angle Q$  and  $\angle R$ ,  $\cos(Q) = \sin(R)$ . The measures, in degrees, of  $\angle Q$  and  $\angle R$  are  $x + 61$  and  $4x + 4$ , respectively. What is the value of  $x$ ?

- A. 5
- B. 19
- C. 23
- D. 29

## ID: dec3599b Answer

Correct Answer: A

### Rationale

Choice A is correct. It's given that for two acute angles,  $\angle Q$  and  $\angle R$ ,  $\cos(Q) = \sin(R)$ . For two acute angles, if the sine of one angle is equal to the cosine of the other angle, the angles are complementary. It follows that  $\angle Q$  and  $\angle R$  are complementary. That is, the sum of the measures of the angles is 90 degrees. It's given that the measure of  $\angle Q$  is  $x + 61$  degrees and the measure of  $\angle R$  is  $4x + 4$  degrees. It follows that  $(x + 61) + (4x + 4) = 90$ . By combining like terms, this equation can be rewritten as  $5x + 65 = 90$ . Subtracting 65 from each side of this equation yields  $5x = 25$ . Dividing each side of this equation by 5 yields  $x = 5$ .

Choice B is incorrect. This would be the value of  $x$  if  $\cos(Q) = \cos(R)$  rather than  $\cos(Q) = \sin(R)$ .

Choice C is incorrect. This would be the value of  $x$  if  $\cos(Q) = -\cos(R)$  rather than  $\cos(Q) = \sin(R)$  and if  $\angle R$  were obtuse rather than acute.

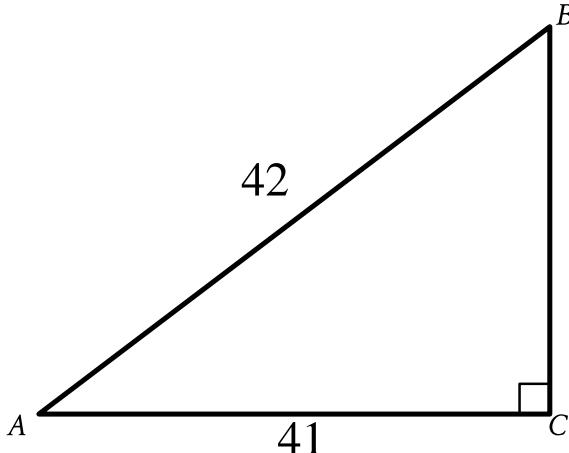
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

# Question ID 8dfac8be

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 8dfac8be



Note: Figure not drawn to scale.

What is the value of  $\cos A$  in the triangle shown?

- A.  $\frac{42}{41}$
- B.  $\frac{41}{42}$
- C.  $\frac{1}{42}$
- D.  $\frac{1}{41}$

ID: 8dfac8be Answer

Correct Answer: B

Rationale

Choice B is correct. The cosine of an acute angle in a right triangle is defined as the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse. In the triangle shown, the length of the leg adjacent to angle  $A$  is 41, and the length of the hypotenuse is 42. Therefore,  $\cos A = \frac{41}{42}$ .

Choice A is incorrect. This is the value of  $\frac{1}{\cos A}$ .

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

# Question ID 63fcb9eb

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

## ID: 63fcb9eb

The length of a rectangle's diagonal is  $5\sqrt{17}$ , and the length of the rectangle's shorter side is 5. What is the length of the rectangle's longer side?

- A.  $\sqrt{17}$
- B. 20
- C.  $15\sqrt{2}$
- D. 400

## ID: 63fcb9eb Answer

Correct Answer: B

### Rationale

Choice B is correct. A rectangle's diagonal divides a rectangle into two congruent right triangles, where the diagonal is the hypotenuse of both triangles. It's given that the length of the diagonal is  $5\sqrt{17}$  and the length of the rectangle's shorter side is 5. Therefore, each of the two right triangles formed by the rectangle's diagonal has a hypotenuse with length  $5\sqrt{17}$ , and a shorter leg with length 5. To calculate the length of the longer leg of each right triangle, the Pythagorean theorem,  $a^2 + b^2 = c^2$ , can be used, where  $a$  and  $b$  are the lengths of the legs and  $c$  is the length of the hypotenuse of the triangle. Substituting 5 for  $a$  and  $5\sqrt{17}$  for  $c$  in the equation  $a^2 + b^2 = c^2$  yields

$5^2 + b^2 = (5\sqrt{17})^2$ , which is equivalent to  $25 + b^2 = 25(17)$ , or  $25 + b^2 = 425$ . Subtracting 25 from each side of this equation yields  $b^2 = 400$ . Taking the positive square root of each side of this equation yields  $b = 20$ . Therefore, the length of the longer leg of each right triangle formed by the diagonal of the rectangle is 20. It follows that the length of the rectangle's longer side is 20.

Choice A is incorrect and may result from dividing the length of the rectangle's diagonal by the length of the rectangle's shorter side, rather than substituting these values into the Pythagorean theorem.

Choice C is incorrect and may result from using the length of the rectangle's diagonal as the length of a leg of the right triangle, rather than the length of the hypotenuse.

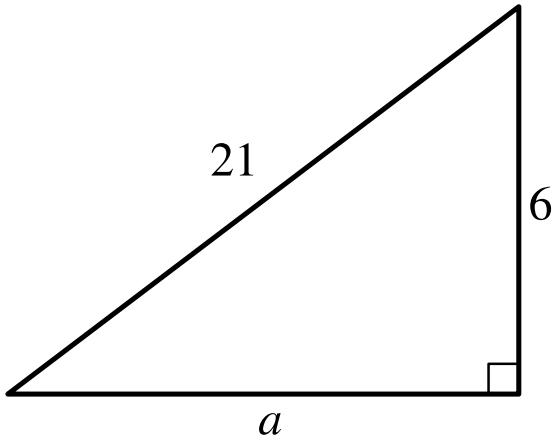
Choice D is incorrect. This is the square of the length of the rectangle's longer side.

Question Difficulty: Hard

# Question ID 66b3574c

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 66b3574c



Note: Figure not drawn to scale.

For the triangle shown, which expression represents the value of  $a$ ?

- A.  $\sqrt{21^2 - 6^2}$
- B.  $21^2 - 6^2$
- C.  $\sqrt{21 - 6}$
- D.  $21 - 6$

ID: 66b3574c Answer

Correct Answer: A

Rationale

Choice A is correct. For the right triangle shown, the lengths of the legs are  $a$  units and 6 units, and the length of the hypotenuse is 21 units. The Pythagorean theorem states that in a right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse. Therefore,  $a^2 + 6^2 = 21^2$ . Subtracting  $6^2$  from both sides of this equation yields  $a^2 = 21^2 - 6^2$ . Taking the square root of both sides of this equation yields  $a = \pm\sqrt{21^2 - 6^2}$ . Since  $a$  is a length,  $a$  must be positive. Therefore,  $a = \sqrt{21^2 - 6^2}$ . Thus, for the triangle shown,  $\sqrt{21^2 - 6^2}$  represents the value of  $a$ .

Choice B is incorrect. For the triangle shown, this expression represents the value of  $a^2$ , not  $a$ .

Choice C is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Hard

# Question ID 0e9cca7f

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

## ID: 0e9cca7f

In triangle  $XYZ$ , angle  $Z$  is a right angle and the length of  $\overline{YZ}$  is 24 units. If  $\tan X = \frac{12}{35}$ , what is the perimeter, in units, of triangle  $XYZ$ ?

- A. 188
- B. 168
- C. 84
- D. 71

## ID: 0e9cca7f Answer

Correct Answer: B

### Rationale

Choice B is correct. It's given that angle  $Z$  in triangle  $XYZ$  is a right angle. Thus, side  $YZ$  is the leg opposite angle  $X$  and side  $XZ$  is the leg adjacent to angle  $X$ . The tangent of an acute angle in a right triangle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. It follows that  $\tan X = \frac{YZ}{XZ}$ . It's given that  $\tan X = \frac{12}{35}$  and the length of side  $YZ$  is 24 units. Substituting  $\frac{12}{35}$  for  $\tan X$  and 24 for  $YZ$  in the equation  $\tan X = \frac{YZ}{XZ}$  yields  $\frac{12}{35} = \frac{24}{XZ}$ . Multiplying both sides of this equation by  $35(XZ)$  yields  $12(XZ) = 24(35)$ , or  $12(XZ) = 840$ . Dividing both sides of this equation by 12 yields  $XZ = 70$ . The length  $XY$  can be calculated using the Pythagorean theorem, which states that if a right triangle has legs with lengths of  $a$  and  $b$  and a hypotenuse with length  $c$ , then  $a^2 + b^2 = c^2$ . Substituting 70 for  $a$  and 24 for  $b$  in this equation yields  $70^2 + 24^2 = c^2$ , or  $5,476 = c^2$ . Taking the square root of both sides of this equation yields  $\pm 74 = c$ . Since the length of the hypotenuse must be positive,  $74 = c$ . Therefore, the length of  $XY$  is 74 units. The perimeter of a triangle is the sum of the lengths of all sides. Thus,  $(74 + 70 + 24)$  units, or 168 units, is the perimeter of triangle  $XYZ$ .

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This would be the perimeter, in units, for a right triangle where the length of side  $YZ$  is 12 units, not 24 units.

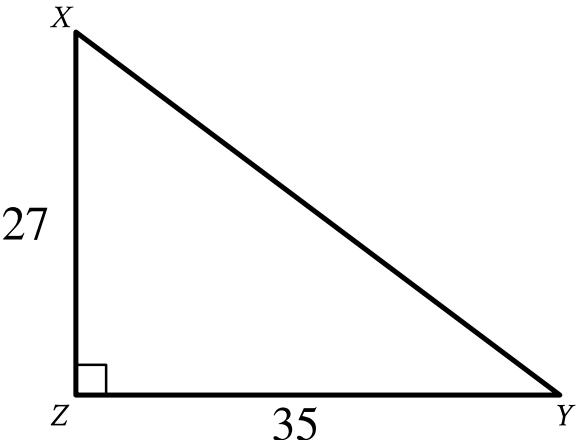
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

# Question ID 878956b3

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 878956b3



Note: Figure not drawn to scale.

Triangle  $XYZ$  shown is a right triangle. Which of the following has the same value as  $\sin X$ ?

- A.  $\tan X$
- B.  $\tan Y$
- C.  $\cos X$
- D.  $\cos Y$

ID: 878956b3 Answer

Correct Answer: D

Rationale

Choice D is correct. The sine of an angle is equal to the cosine of its complementary angle. In the triangle shown, angle  $Z$  is a right angle; thus, angles  $X$  and  $Y$  are complementary angles. Therefore,  $\cos Y$  has the same value as  $\sin X$ .

Choice A is incorrect and may result from conceptual errors.

Choice B is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Question Difficulty: Hard

## Question ID 23fc2b60

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 23fc2b60

In  $\triangle ABC$ ,  $\angle B$  is a right angle and the length of  $\overline{BC}$  is 136 millimeters. If  $\cos A = \frac{3}{5}$ , what is the length, in millimeters, of  $\overline{AB}$ ?

- A. 34
- B. 102
- C. 136
- D. 170

ID: 23fc2b60 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that in  $\triangle ABC$ ,  $\angle B$  is a right angle. Therefore,  $\triangle ABC$  is a right triangle, and  $\overline{AC}$  is the hypotenuse of the triangle. It's also given that  $\cos A = \frac{3}{5}$ . Since the cosine of an acute angle in a right triangle is defined as the ratio of the length of the side adjacent to the angle to the length of the hypotenuse, the ratio of the length of  $\overline{AB}$  to the length of  $\overline{AC}$  is 3 to 5. It follows that the length of  $\overline{AB}$  can be represented as  $3a$  and the length of  $\overline{AC}$  can be represented as  $5a$ , where  $a$  is a constant. The Pythagorean theorem states that the sum of the squares of the length of the legs of a right triangle is equal to the square of the length of its hypotenuse, so it follows that  $AB^2 + BC^2 = AC^2$ . Substituting  $3a$  for  $AB$  and  $5a$  for  $AC$  in this equation yields  $(3a)^2 + BC^2 = (5a)^2$ , or  $9a^2 + BC^2 = 25a^2$ . Subtracting  $9a^2$  from both sides of this equation yields  $BC^2 = 16a^2$ , or  $BC = 4a$ . It follows that the ratio of the length of  $\overline{AB}$  to the length of  $\overline{BC}$  is 3 to 4. Let  $x$  represent the length, in millimeters, of  $\overline{AB}$ . It's given that the length of  $\overline{BC}$  is 136 millimeters. Since the ratio of the length of  $\overline{AB}$  to the length of  $\overline{BC}$  is 3 to 4,  $\frac{x}{136} = \frac{3}{4}$ . Multiplying both sides of this equation by 136 yields  $x = \frac{3(136)}{4}$ , or  $x = 102$ . Therefore, the length of  $\overline{AB}$  is 102 millimeters.

Choice A is incorrect. This is the scale factor by which the 3 to 4 to 5 ratio is multiplied that results in the side lengths of  $\triangle ABC$ .

Choice C is incorrect. This is the length of  $\overline{BC}$ , not the length of  $\overline{AB}$ .

Choice D is incorrect. This is the length of  $\overline{AC}$ , not the length of  $\overline{AB}$ .

Question Difficulty: Hard

# Question ID 331017d5

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

**ID: 331017d5**

A right triangle has legs with lengths of **28** centimeters and **20** centimeters. What is the length of this triangle's hypotenuse, in centimeters?

- A.  $8\sqrt{6}$
- B.  $4\sqrt{74}$
- C. 48
- D. 1,184

**ID: 331017d5 Answer**

Correct Answer: B

Rationale

Choice B is correct. The Pythagorean theorem states that in a right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse. It's given that the right triangle has legs with lengths of **28** centimeters and **20** centimeters. Let  $c$  represent the length of this triangle's hypotenuse, in centimeters. Therefore, by the Pythagorean theorem,  $28^2 + 20^2 = c^2$ , or  $1,184 = c^2$ . Taking the positive square root of both sides of this equation yields  $\sqrt{1,184} = c$ , or  $4\sqrt{74} = c$ . Therefore, the length of this triangle's hypotenuse, in centimeters, is  $4\sqrt{74}$ .

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This is the square of the length of the triangle's hypotenuse.

Question Difficulty: Hard

# Question ID 08db26b7

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

## ID: 08db26b7

The perimeter of an isosceles right triangle is  $18 + 18\sqrt{2}$  inches. What is the length, in inches, of the hypotenuse of this triangle?

- A. 9
- B.  $9\sqrt{2}$
- C. 18
- D.  $18\sqrt{2}$

## ID: 08db26b7 Answer

Correct Answer: C

### Rationale

Choice C is correct. The perimeter of a triangle is the sum of the lengths of its sides. Since the given triangle is an isosceles right triangle, the length of each leg is the same and the length of the hypotenuse is equal to  $\sqrt{2}$  times the length of a leg. Let  $x$  represent the length, in inches, of a leg of this isosceles right triangle. Therefore, the perimeter, in inches, of the triangle is  $x + x + x\sqrt{2}$ , or  $2x + x\sqrt{2}$ , which is equivalent to  $x(2 + \sqrt{2})$ . It's given that the perimeter of this triangle is  $18 + 18\sqrt{2}$  inches. Thus,  $x(2 + \sqrt{2}) = 18 + 18\sqrt{2}$ . Dividing both sides of this equation by  $2 + \sqrt{2}$  yields  $x = \frac{18+18\sqrt{2}}{2+\sqrt{2}}$ . Multiplying the right-hand side of this equation by  $\frac{2-\sqrt{2}}{2-\sqrt{2}}$  yields  $x = \frac{36+36\sqrt{2}-18\sqrt{2}-36}{2}$ , or  $x = 9\sqrt{2}$ . It follows that the length, in inches, of a leg of this isosceles right triangle is  $9\sqrt{2}$ . Therefore, the length, in inches, of the hypotenuse of this isosceles right triangle is  $(9\sqrt{2})(\sqrt{2})$ , or 18.

Choice A is incorrect. If this were the length of the hypotenuse, the perimeter would be  $9 + 9\sqrt{2}$  inches.

Choice B is incorrect. This is the length, in inches, of a leg of this triangle, not the hypotenuse.

Choice D is incorrect. If this were the length of the hypotenuse, the perimeter would be  $36 + 18\sqrt{2}$  inches.

Question Difficulty: Hard

# Question ID 0ef6cdb0

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

**ID: 0ef6cdb0**

A right triangle has legs with lengths of **24** centimeters and **21** centimeters. If the length of this triangle's hypotenuse, in centimeters, can be written in the form  $3\sqrt{d}$ , where  $d$  is an integer, what is the value of  $d$ ?

**ID: 0ef6cdb0 Answer**

Correct Answer: 113

Rationale

The correct answer is **113**. It's given that the legs of a right triangle have lengths **24** centimeters and **21** centimeters. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs. It follows that if  $h$  represents the length, in centimeters, of the hypotenuse of the right triangle,  $h^2 = 24^2 + 21^2$ . This equation is equivalent to  $h^2 = 1,017$ . Taking the square root of each side of this equation yields  $h = \sqrt{1,017}$ . This equation can be rewritten as  $h = \sqrt{9 \cdot 113}$ , or  $h = \sqrt{9} \cdot \sqrt{113}$ . This equation is equivalent to  $h = 3\sqrt{113}$ . It's given that the length of the triangle's hypotenuse, in centimeters, can be written in the form  $3\sqrt{d}$ . It follows that the value of  $d$  is **113**.

Question Difficulty: Hard

## Question ID 02210929

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 02210929

Which of the following expressions is equivalent to  $(\sin 24^\circ)(\cos 66^\circ) + (\cos 24^\circ)(\sin 66^\circ)$ ?

- A.  $2(\cos 66^\circ)(\sin 24^\circ)$
- B.  $2(\cos 66^\circ) + 2(\cos 24^\circ)$
- C.  $(\cos 66^\circ)^2 + (\cos 24^\circ)^2$
- D.  $(\cos 66^\circ)^2 + (\sin 24^\circ)^2$

ID: 02210929 Answer

Correct Answer: C

Rationale

Choice C is correct. The sine of an angle is equal to the cosine of its complementary angle. Since angles with measures  $24^\circ$  and  $66^\circ$  are complementary to each other,  $\sin 24^\circ$  is equal to  $\cos 66^\circ$  and  $\sin 66^\circ$  is equal to  $\cos 24^\circ$ . Substituting  $\cos 66^\circ$  for  $\sin 24^\circ$  and  $\cos 24^\circ$  for  $\sin 66^\circ$  in the given expression yields  $(\cos 66^\circ)(\cos 66^\circ) + (\cos 24^\circ)(\cos 24^\circ)$ , or  $(\cos 66^\circ)^2 + (\cos 24^\circ)^2$ .

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

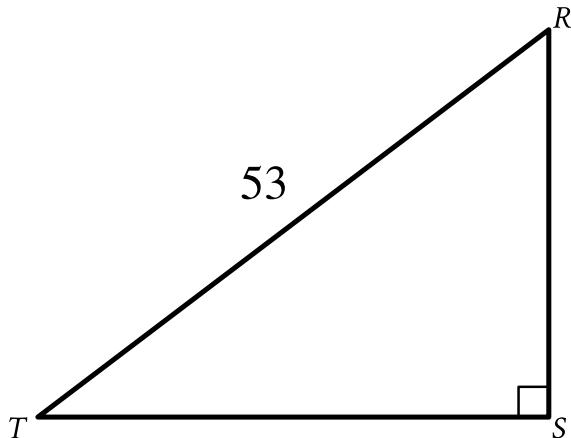
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

# Question ID d34d8b08

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: d34d8b08



Note: Figure not drawn to scale.

In the triangle shown,  $RS = \sqrt{105}$ . What is the value of  $\sin R$ ?

ID: d34d8b08 Answer

Correct Answer: .9811, 52/53

Rationale

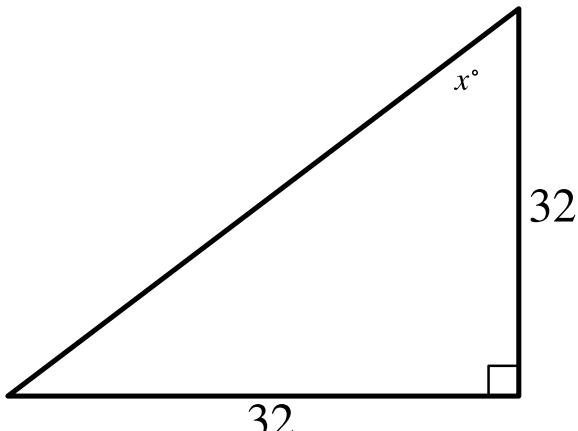
The correct answer is  $\frac{52}{53}$ . In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs. The length of the hypotenuse of the right triangle shown is 53. It's given that  $RS = \sqrt{105}$ . Therefore, the length of one of the legs of the triangle shown is  $\sqrt{105}$ . Let  $x$  represent  $TS$ , the length of the other leg of the triangle shown. Therefore,  $53^2 = (\sqrt{105})^2 + x^2$ , or  $2,809 = 105 + x^2$ . Subtracting 105 from both sides of this equation yields  $2,704 = x^2$ . Taking the positive square root of both sides of this equation yields  $52 = x$ . Therefore,  $TS$ , the length of the other leg of the triangle shown, is 52. The sine of an acute angle in a right triangle is defined as the ratio of the length of the leg opposite the angle to the length of the hypotenuse. The length of the leg opposite angle  $R$  is 52, and the length of the hypotenuse is 53. Therefore, the value of  $\sin R$  is  $\frac{52}{53}$ . Note that 52/53 or .9811 are examples of ways to enter a correct answer.

Question Difficulty: Hard

# Question ID e292a69b

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: e292a69b



Note: Figure not drawn to scale.

In the triangle shown, what is the value of  $x$ ?

ID: e292a69b Answer

Correct Answer: 45

Rationale

The correct answer is 45. An isosceles right triangle has a right angle and two legs of equal length. In the triangle shown, one angle is a right angle and the two legs each have a length of 32. Thus, the given triangle is an isosceles right triangle. In an isosceles right triangle, the measures of the two non-right angles are  $45^\circ$ . It follows that the value of  $x$  is 45.

Question Difficulty: Hard

# Question ID 3740eab2

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 3740eab2

In triangle  $ABC$ , angle  $B$  is a right angle. The length of side  $AB$  is  $10\sqrt{37}$  and the length of side  $BC$  is  $24\sqrt{37}$ . What is the length of side  $AC$ ?

- A.  $14\sqrt{37}$
- B.  $26\sqrt{37}$
- C.  $34\sqrt{37}$
- D.  $\sqrt{34 \cdot 37}$

ID: 3740eab2 Answer

Correct Answer: B

Rationale

Choice B is correct. The Pythagorean theorem states that for a right triangle,  $c^2 = a^2 + b^2$ , where  $c$  represents the length of the hypotenuse and  $a$  and  $b$  represent the lengths of the legs. It's given that in triangle  $ABC$ , angle  $B$  is a right angle. Therefore, triangle  $ABC$  is a right triangle, where the hypotenuse is side  $AC$  and the legs are sides  $AB$  and  $BC$ . It's given that the lengths of sides  $AB$  and  $BC$  are  $10\sqrt{37}$  and  $24\sqrt{37}$ , respectively. Substituting these values for  $a$  and  $b$  in the formula  $c^2 = a^2 + b^2$  yields  $c^2 = (10\sqrt{37})^2 + (24\sqrt{37})^2$ , which is equivalent to  $c^2 = 100(37) + 576(37)$ , or  $c^2 = 676(37)$ . Taking the square root of both sides of this equation yields  $c = \pm 26\sqrt{37}$ . Since  $c$  represents the length of the hypotenuse, side  $AC$ ,  $c$  must be positive. Therefore, the length of side  $AC$  is  $26\sqrt{37}$ .

Choice A is incorrect. This is the result of solving the equation  $c = 24\sqrt{37} - 10\sqrt{37}$ , not  $c^2 = (10\sqrt{37})^2 + (24\sqrt{37})^2$ .

Choice C is incorrect. This is the result of solving the equation  $c = 10\sqrt{37} + 24\sqrt{37}$ , not  $c^2 = (10\sqrt{37})^2 + (24\sqrt{37})^2$ .

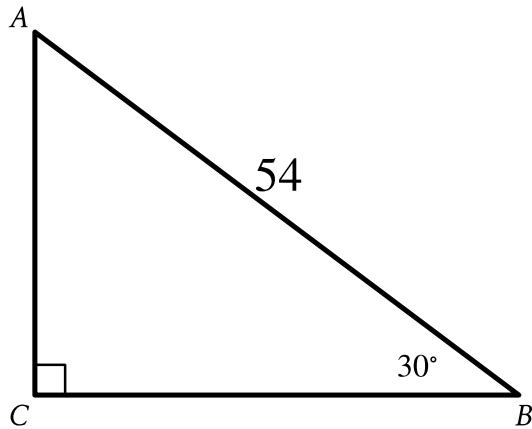
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

# Question ID 309cf71a

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 309cf71a



Note: Figure not drawn to scale.

Right triangle  $ABC$  is shown. What is the value of  $\tan A$ ?

- A.  $\frac{\sqrt{3}}{54}$
- B.  $\frac{1}{\sqrt{3}}$
- C.  $\sqrt{3}$
- D.  $27\sqrt{3}$

ID: 309cf71a Answer

Correct Answer: C

Rationale

Choice C is correct. In the triangle shown, the measure of angle  $B$  is  $30^\circ$  and angle  $C$  is a right angle, which means that it has a measure of  $90^\circ$ . Since the sum of the angles in a triangle is equal to  $180^\circ$ , the measure of angle  $A$  is equal to  $180^\circ - (30 + 90)^\circ$ , or  $60^\circ$ . In a right triangle whose acute angles have measures  $30^\circ$  and  $60^\circ$ , the lengths of the legs can be represented by the expressions  $x$ ,  $x\sqrt{3}$ , and  $2x$ , where  $x$  is the length of the leg opposite the angle with measure  $30^\circ$ ,  $x\sqrt{3}$  is the length of the leg opposite the angle with measure  $60^\circ$ , and  $2x$  is the length of the hypotenuse. In the triangle shown, the hypotenuse has a length of 54. It follows that  $2x = 54$ , or  $x = 27$ . Therefore, the length of the leg opposite angle  $B$  is 27 and the length of the leg opposite angle  $A$  is  $27\sqrt{3}$ . The tangent of an acute angle in a right triangle is defined as the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. The length of the leg opposite angle  $A$  is  $27\sqrt{3}$  and the length of the leg adjacent to angle  $A$  is 27. Therefore, the value of  $\tan A$  is  $\frac{27\sqrt{3}}{27}$ , or  $\sqrt{3}$ .

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect. This is the value of  $\frac{1}{\tan A}$ , not the value of  $\tan A$ .

Choice D is incorrect. This is the length of the leg opposite angle  $A$ , not the value of  $\tan A$ .

Question Difficulty: Hard