

Question ID ba6b299f

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: ba6b299f

The quadratic function g models the depth, in meters, below the surface of the water of a seal t minutes after the seal entered the water during a dive. The function estimates that the seal reached its maximum depth of **302.4** meters **6** minutes after it entered the water and then reached the surface of the water **12** minutes after it entered the water. Based on the function, what was the estimated depth, to the nearest meter, of the seal **10** minutes after it entered the water?

ID: ba6b299f Answer

Correct Answer: 168

Rationale

The correct answer is **168**. The quadratic function g gives the estimated depth of the seal, $g(t)$, in meters, t minutes after the seal enters the water. It's given that function g estimates that the seal reached its maximum depth of **302.4** meters **6** minutes after it entered the water. Therefore, function g can be expressed in vertex form as $g(t) = a(t - 6)^2 + 302.4$, where a is a constant. Since it's also given that the seal reached the surface of the water after **12** minutes, $g(12) = 0$. Substituting **12** for t and **0** for $g(t)$ in $g(t) = a(t - 6)^2 + 302.4$ yields $0 = a(12 - 6)^2 + 302.4$, or $36a = -302.4$. Dividing both sides of this equation by **36** gives $a = -8.4$. Substituting -8.4 for a in $g(t) = a(t - 6)^2 + 302.4$ gives $g(t) = -8.4(t - 6)^2 + 302.4$. Substituting **10** for t in $g(t)$ gives $g(10) = -8.4(10 - 6)^2 + 302.4$, which is equivalent to $g(10) = -8.4(4)^2 + 302.4$, or $g(10) = 168$. Therefore, the estimated depth, to the nearest meter, of the seal **10** minutes after it entered the water was **168** meters.

Question Difficulty: Hard

Question ID e61cf654

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: e61cf654

The function f is defined by $f(x) = |x - 4x|$. What value of a satisfies $f(5) - f(a) = -15$?

- A. -20
- B. 5
- C. 10
- D. 45

ID: e61cf654 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the function f is defined by $f(x) = |x - 4x|$. It's also given that $f(5) - f(a) = -15$. Substituting 5 for x in the function $f(x) = |x - 4x|$ yields $f(5) = |5 - 4(5)|$ and substituting a for x in the function $f(x) = |x - 4x|$ yields $f(a) = |a - 4a|$. Therefore, $f(5) = 15$ and $f(a) = |-3a|$. Substituting 15 for $f(5)$ and $|-3a|$ for $f(a)$ in the equation $f(5) - f(a) = -15$ yields $15 - |-3a| = -15$. Subtracting 15 from both sides of this equation yields $-|-3a| = -30$. Dividing both sides of this equation by -1 yields $|-3a| = 30$. By the definition of absolute value, if $|-3a| = 30$, then $-3a = 30$ or $-3a = -30$. Dividing both sides of each of these equations by -3 yields $a = -10$ or $a = 10$, respectively. Thus, of the given choices, a value of a that satisfies $f(5) - f(a) = -15$ is 10 .

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID fe023b12

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: fe023b12

$y = 576^{(2x+2)}$

The graph of the given equation in the xy -plane has a y -intercept of (r, s) . Which of the following equivalent equations displays the value of s as a constant, a coefficient, or the base?

- A. $y = \text{msup}$
- B. $y = \text{msup}$
- C. $y = \frac{1}{24} \text{msup}$
- D. $y = \frac{1}{576} \text{msup}$

ID: fe023b12 Answer

Correct Answer: A

Rationale

Choice A is correct. The y -intercept of a graph in the xy -plane is the point where $x = 0$. Substituting 0 for x in the given equation, $y = 576^{(2x+2)}$, yields $y = 576^{(2(0)+2)}$, which is equivalent to $y = 576^2$, or $y = 331,776$. Therefore, the graph of the given equation in the xy -plane has a y -intercept of $(0, 331,776)$. It follows that $r = 0$ and $s = 331,776$. Thus, the equivalent equation $y = 331,776^{(x+1)}$ displays the value of s as the base.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 89aff4d3

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 89aff4d3

At the time that an article was first featured on the home page of a news website, there were 40 comments on the article. An exponential model estimates that at the end of each hour after the article was first featured on the home page, the number of comments on the article had increased by 190% of the number of comments on the article at the end of the previous hour. Which of the following equations best represents this model, where C is the estimated number of comments on the article t hours after the article was first featured on the home page and $t \leq 4$?

- A. $C = 40^{msup}$
- B. $C = 40^{msup}$
- C. $C = 40^{msup}$
- D. $C = 40^{msup}$

ID: 89aff4d3 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that an exponential model estimates that the number of comments on an article increased by a fixed percentage at the end of each hour. Therefore, the model can be represented by an exponential equation of the form $C = Ka^t$, where C is the estimated number of comments on the article t hours after the article was first featured on the home page and K and a are constants. It's also given that when the article was first featured on the home page of the news website, there were 40 comments on the article. This means that when $t = 0$, $C = 40$. Substituting 0 for t and 40 for C in the equation $C = Ka^t$ yields $40 = Ka^0$, or $40 = K$. It's also given that the number of comments on the article at the end of an hour had increased by 190% of the number of comments on the article at the end of the previous hour. Multiplying the percent increase by the number of comments on the article at the end of the previous hour yields the number of estimated additional comments the article has on its home page: $(40)(\frac{190}{100})$, or 76 comments. Thus, the estimated number of comments for the following hour is the sum of the comments from the end of the previous hour and the number of additional comments, which is $40 + 76$, or 116. This means that when $t = 1$, $C = 116$. Substituting 1 for t , 116 for C , and 40 for K in the equation $C = Ka^t$ yields $116 = 40a^1$, or $116 = 40a$. Dividing both sides of this equation by 40 yields $2.9 = a$. Substituting 40 for K and 2.9 for a in the equation $C = Ka^t$ yields $C = 40(2.9)^t$. Thus, the equation that best represents this model is $C = 40(2.9)^t$.

Choice A is incorrect. This model represents a situation where the number of comments at the end of each hour increased by 19% of the number of comments at the end of the previous hour, rather than 190%.

Choice B is incorrect. This model represents a situation where the number of comments at the end of each hour increased by 90% of the number of comments at the end of the previous hour, rather than 190%.

Choice C is incorrect. This model represents a situation where the number of comments at the end of each hour was 19 times the number of comments at the end of the previous hour, rather than increasing by 190% of the number of comments at the end of the previous hour.

Question Difficulty: Hard

Question ID f4ae2eb4

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: f4ae2eb4

Which of the following functions has(have) a minimum value at -3 ?

I. $f(x) = -6(3)^x - 3$

II. $g(x) = -3(6)^x$

- A. I only
- B. II only
- C. I and II
- D. Neither I nor II

ID: f4ae2eb4 Answer

Correct Answer: D

Rationale

Choice D is correct. A function of the form $f(x) = a(b)^x + c$, where $a < 0$ and $b > 1$, is a decreasing function. Both of the given functions are of this form; therefore, both are decreasing functions. If a function f is decreasing as the value of x increases, the corresponding value of $f(x)$ decreases; therefore, the function doesn't have a minimum value. Thus, neither of the given functions has a minimum value.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID a5ff934b

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: a5ff934b

The area of a triangle is equal to x^2 square centimeters. The length of the base of the triangle is $2x + 22$ centimeters, and the height of the triangle is $x - 10$ centimeters. What is the value of x ?

ID: a5ff934b Answer

Correct Answer: 110

Rationale

The correct answer is **110**. The area of a triangle is equal to one half of the product of the length of the base of the triangle and the height of the triangle. It's given that the length of the base of the triangle is $2x + 22$ centimeters and the height of the triangle is $x - 10$ centimeters; therefore, its area is $\frac{1}{2}(2x + 22)(x - 10)$ square centimeters. It's also given that the area of the triangle is equal to x^2 square centimeters. Therefore, it follows that $\frac{1}{2}(2x + 22)(x - 10) = x^2$. This equation can be rewritten as $(x + 11)(x - 10) = x^2$, or $x^2 + x - 110 = x^2$. Subtracting x^2 from both sides of this equation yields $x - 110 = 0$. Adding **110** to both sides of this equation yields $x = \mathbf{110}$. Therefore, the value of x is **110**.

Question Difficulty: Hard

Question ID dd31281d

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: dd31281d

$$P(t) = 260(1.04)^{(\frac{6}{4})t}$$

The function P models the population, in thousands, of a certain city t years after **2003**. According to the model, the population is predicted to increase by **4%** every n months. What is the value of n ?

- A. 8
- B. 12
- C. 18
- D. 72

ID: dd31281d Answer

Correct Answer: A

Rationale

Choice A is correct. It’s given that the function P models the population, in thousands, of a certain city t years after **2003**. The value of the base of the given exponential function, **1.04**, corresponds to an increase of **4%** for every increase of **1** in the exponent, $(\frac{6}{4})t$. If the exponent is equal to **0**, then $(\frac{6}{4})t = 0$. Multiplying both sides of this equation by $(\frac{4}{6})$ yields $t = 0$. If the exponent is equal to **1**, then $(\frac{6}{4})t = 1$. Multiplying both sides of this equation by $(\frac{4}{6})$ yields $t = \frac{4}{6}$, or $t = \frac{2}{3}$. Therefore, the population is predicted to increase by **4%** every $\frac{2}{3}$ of a year. It’s given that the population is predicted to increase by **4%** every n months. Since there are **12** months in a year, $\frac{2}{3}$ of a year is equivalent to $(\frac{2}{3})(12)$, or **8**, months. Therefore, the value of n is **8**.

Choice B is incorrect. This is the number of months in which the population is predicted to increase by **4%** according to the model $P(t) = 260(1.04)^t$, not $P(t) = 260(1.04)^{(\frac{6}{4})t}$.

Choice C is incorrect. This is the number of months in which the population is predicted to increase by **4%** according to the model $P(t) = 260(1.04)^{(\frac{4}{6})t}$, not $P(t) = 260(1.04)^{(\frac{6}{4})t}$.

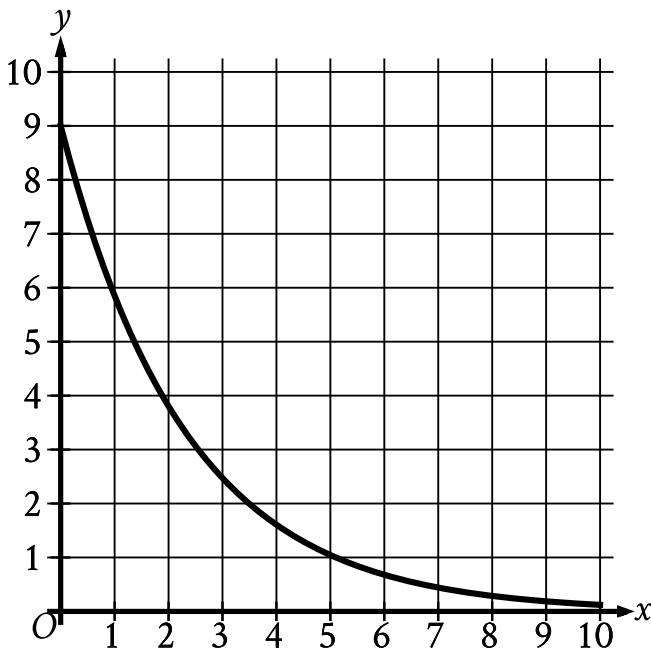
Choice D is incorrect. This is the number of months in which the population is predicted to increase by **4%** according to the model $P(t) = 260(1.04)^{(\frac{1}{6})t}$, not $P(t) = 260(1.04)^{(\frac{6}{4})t}$.

Question Difficulty: Hard

Question ID 1f8d25fc

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 1f8d25fc



The graph gives the estimated number of catalogs y , in thousands, a company sent to its customers at the end of each year, where x represents the number of years since the end of **1992**, where $0 \leq x \leq 10$. Which statement is the best interpretation of the y -intercept in this context?

- A. The estimated total number of catalogs the company sent to its customers during the first **10** years was **9,000**.
- B. The estimated total number of catalogs the company sent to its customers from the end of **1992** to the end of **2002** was **90**.
- C. The estimated number of catalogs the company sent to its customers at the end of **1992** was **9**.
- D. The estimated number of catalogs the company sent to its customers at the end of **1992** was **9,000**.

ID: 1f8d25fc Answer

Correct Answer: D

Rationale

Choice D is correct. The y -intercept of the graph is the point at which the graph crosses the y -axis, or the point for which the value of x is **0**. Therefore, the y -intercept of the given graph is the point **(0, 9)**. It's given that x represents the number of years since the end of **1992**. Therefore, $x = 0$ represents **0** years since the end of **1992**, which is the same as the end of **1992**. It's also given that y represents the estimated number of catalogs, in thousands, that the company sent to its customers at the end of the year. Therefore, $y = 9$ represents **9,000** catalogs. It follows that the y -intercept **(0, 9)** means that the estimated number of catalogs the company sent to its customers at the end of **1992** was **9,000**.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 97e5f390

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 97e5f390

The product of two positive integers is **462**. If the first integer is **5** greater than twice the second integer, what is the smaller of the two integers?

ID: 97e5f390 Answer

Correct Answer: 14

Rationale

The correct answer is **14**. Let x represent the first integer and y represent the second integer. If the first integer is **5** greater than twice the second integer, then $x = 2y + 5$. It's given that the product of the two integers is **462**; therefore $xy = 462$. Substituting $2y + 5$ for x in this equation yields $(2y + 5)(y) = 462$, which can be written as $2y^2 + 5y = 462$. Subtracting **462** from each side of this equation yields $2y^2 + 5y - 462 = 0$. The left-hand side of this equation can be factored by finding two values whose product is $2(-462)$, or -924 , and whose sum is **5**. The two values whose product is -924 and whose sum is **5** are **33** and -28 . Thus, the equation $2y^2 + 5y - 462 = 0$ can be rewritten as $2y^2 - 28y + 33y - 462 = 0$, which is equivalent to $2y(y - 14) + 33(y - 14) = 0$, or $(2y + 33)(y - 14) = 0$. By the zero product property, it follows that $2y + 33 = 0$ or $y - 14 = 0$. Subtracting **33** from both sides of the equation $2y + 33 = 0$ yields $2y = -33$. Dividing both sides of this equation by **2** yields $y = -\frac{33}{2}$. Since y is a positive integer, the value of y isn't $-\frac{33}{2}$. Adding **14** to both sides of the equation $y - 14 = 0$ yields $y = 14$. Substituting **14** for y in the equation $xy = 462$ yields $x(14) = 462$. Dividing both sides of this equation by **14** yields $x = 33$. Therefore, the two integers are **14** and **33**, so the smaller of the two integers is **14**.

Question Difficulty: Hard

Question ID bb19a769

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: bb19a769

$$f(x) = (x - 2)(x + 15)$$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

ID: bb19a769 Answer

Correct Answer: -13/2, -6.5

Rationale

The correct answer is $-\frac{13}{2}$. The value of x for which $f(x)$ reaches its minimum can be found by rewriting the given equation in the form $f(x) = (x - h)^2 + k$, where $f(x)$ reaches its minimum, k , when the value of x is h . The given equation, $f(x) = (x - 2)(x + 15)$, can be rewritten as $f(x) = x^2 + 13x - 30$. By completing the square, this equation can be rewritten as $f(x) = \left(x^2 + 13x + \left(\frac{13}{2}\right)^2\right) - 30 - \left(\frac{13}{2}\right)^2$, which is equivalent to $f(x) = \left(x + \frac{13}{2}\right)^2 - \frac{289}{4}$, or $f(x) = \left(x - \left(-\frac{13}{2}\right)\right)^2 - \frac{289}{4}$. Therefore, $f(x)$ reaches its minimum when the value of x is $-\frac{13}{2}$. Note that -13/2 and -6.5 are examples of ways to enter a correct answer.

Alternate approach: The graph of $y = f(x)$ in the xy -plane is a parabola. The value of x for the vertex of a parabola is the x -value of the midpoint between the two x -intercepts of the parabola. Since it's given that $f(x) = (x - 2)(x + 15)$, it follows that the two x -intercepts of the graph of $y = f(x)$ in the xy -plane occur when $x = 2$ and $x = -15$, or at the points $(2, 0)$ and $(-15, 0)$. The midpoint between two points, (x_1, y_1) and (x_2, y_2) , is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. Therefore, the midpoint between $(2, 0)$ and $(-15, 0)$ is $\left(\frac{2 - 15}{2}, \frac{0 + 0}{2}\right)$, or $\left(-\frac{13}{2}, 0\right)$. It follows that $f(x)$ reaches its minimum when the value of x is $-\frac{13}{2}$. Note that -13/2 and -6.5 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 59c7ac54

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 59c7ac54

$$f(x) = 4,000(0.75)^x$$

An entomologist recommended a program to reduce a certain invasive beetle population in an area. The given function estimates this beetle species' population x years after **2012**, where $x \leq 7$. Which of the following is the best interpretation of **4,000** in this context?

- A. The estimated initial beetle population for this species and area in **2012**
- B. The estimated beetle population for this species and area **7** years after **2012**
- C. The estimated percent decrease in the beetle population for this species and area each year after **2012**
- D. The estimated percent decrease in the beetle population for this species and area every **7** years after **2012**

ID: 59c7ac54 Answer

Correct Answer: A

Rationale

Choice A is correct. For an exponential function in the form $f(x) = a(b)^x$, where a and b are positive constants and $b < 1$, the initial value of $f(x)$, or the value of $f(x)$ when $x = 0$, is a and the value of $f(x)$ decreases by $100(1 - b)\%$ each time x increases by 1 . Therefore, the initial value of the function $f(x) = 4,000(0.75)^x$, or the value of $f(x)$ when $x = 0$, is **4,000**. Therefore, the best interpretation of **4,000** in this context is the estimated initial beetle population for this species and area in **2012**.

Choice B is incorrect. The estimated beetle population for this species and area **7** years after **2012** is $4,000(0.75)^7$, or approximately **534**, not **4,000**.

Choice C is incorrect. The estimated percent decrease in the beetle population for this species and area each year after **2012** is $100(1 - 0.75)$, or **25**, not **4,000**.

Choice D is incorrect. The estimated percent decrease in the beetle population for this species and area every **7** years after **2012** is $100(1 - 0.75^7)$, or approximately **87**, not **4,000**.

Question Difficulty: Hard

Question ID 40dad627

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 40dad627

$$f(x) = 9,000(0.66)^x$$

The given function f models the number of advertisements a company sent to its clients each year, where x represents the number of years since **1997**, and $0 \leq x \leq 5$. If $y = f(x)$ is graphed in the xy -plane, which of the following is the best interpretation of the y -intercept of the graph in this context?

- A. The minimum estimated number of advertisements the company sent to its clients during the **5** years was **1,708**.
- B. The minimum estimated number of advertisements the company sent to its clients during the **5** years was **9,000**.
- C. The estimated number of advertisements the company sent to its clients in **1997** was **1,708**.
- D. The estimated number of advertisements the company sent to its clients in **1997** was **9,000**.

ID: 40dad627 Answer

Correct Answer: D

Rationale

Choice D is correct. The y -intercept of a graph in the xy -plane is the point where $x = 0$. For the given function f , the y -intercept of the graph of $y = f(x)$ in the xy -plane can be found by substituting **0** for x in the equation $y = 9,000(0.66)^x$, which gives $y = 9,000(0.66)^0$. This is equivalent to $y = 9,000(1)$, or $y = 9,000$. Therefore, the y -intercept of the graph of $y = f(x)$ is **(0, 9,000)**. It's given that the function f models the number of advertisements a company sent to its clients each year. Therefore, $f(x)$ represents the estimated number of advertisements the company sent to its clients each year. It's also given that x represents the number of years since **1997**. Therefore, $x = 0$ represents **0** years since **1997**, or **1997**. Thus, the best interpretation of the y -intercept of the graph of $y = f(x)$ is that the estimated number of advertisements the company sent to its clients in **1997** was **9,000**.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID d809f2e5

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: d809f2e5

A right rectangular prism has a height of **9** inches. The length of the prism's base is x inches, which is **7** inches more than the width of the prism's base. Which function V gives the volume of the prism, in cubic inches, in terms of the length of the prism's base?

- A. $V(x) = x(x + 9)(x + 7)$
- B. $V(x) = x(x + 9)(x - 7)$
- C. $V(x) = 9x(x + 7)$
- D. $V(x) = 9x(x - 7)$

ID: d809f2e5 Answer

Correct Answer: D

Rationale

Choice D is correct. The volume of a right rectangular prism can be represented by a function V that gives the volume of the prism, in cubic inches, in terms of the length of the prism's base. The volume of a right rectangular prism is equal to the area of its base times its height. It's given that the length of the prism's base is x inches, which is **7** inches more than the width of the prism's base. This means that the width of the prism's base is $x - 7$ inches. It follows that the area of the prism's base, in square inches, is $x(x - 7)$ and the volume, in cubic inches, of the prism is $x(x - 7)(9)$. Thus, the function V that gives the volume of this right rectangular prism, in cubic inches, in terms of the length of the prism's base, x , is $V(x) = 9x(x - 7)$.

Choice A is incorrect. This function would give the volume of the prism if the height were **9** inches more than the length of its base and the width of the base were **7** inches more than its length.

Choice B is incorrect. This function would give the volume of the prism if the height were **9** inches more than the length of its base.

Choice C is incorrect. This function would give the volume of the prism if the width of the base were **7** inches more than its length, rather than the length of the base being **7** inches more than its width.

Question Difficulty: Hard

Question ID 55cc3589

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 55cc3589

$f(t) = 55t - 2t^2$

The function f is defined by the given equation. The function g is defined by $g(t) = f(t) + 3$. Which expression represents the maximum value of $g(t)$?

- A. $3 +$ ^{msup}
- B. $3 + 2$ ^{msup}
- C. $3 - 2$ ^{msup}
- D. $3 -$ ^{msup}

ID: 55cc3589 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that function g is defined by $g(t) = f(t) + 3$ and that $f(t) = 55t - 2t^2$. Substituting $55t - 2t^2$ for $f(t)$ in $g(t) = f(t) + 3$ yields $g(t) = 55t - 2t^2 + 3$, or $g(t) = -2t^2 + 55t + 3$. The maximum value of $g(t)$ can be found by completing the square to rewrite the equation defining g in the form $g(t) = a(t - h)^2 + k$, where the maximum value of the function is k , which occurs when $t = h$, and a is a negative constant. The equation $g(t) = -2t^2 + 55t + 3$ is equivalent to $g(t) = -2(t^2 - \frac{55}{2}t) + 3$, which can be rewritten as $g(t) = -2(t^2 - \frac{55}{2}t + (\frac{55}{4})^2) + 3 + 2(\frac{55}{4})^2$, or $g(t) = -2(t - \frac{55}{4})^2 + 3 + 2(\frac{55}{4})^2$. This equation is in the form $g(t) = a(t - h)^2 + k$, where $a = -2$, $h = \frac{55}{4}$, and $k = 3 + 2(\frac{55}{4})^2$. Thus, the maximum value of $g(t)$ is $3 + 2(\frac{55}{4})^2$.

Alternate approach: Since the function f is a quadratic function, the maximum value of $f(t)$ occurs at the value of t that's halfway between the two zeros of the function. The zeros of function f can be found by substituting 0 for $f(t)$ in the equation defining f , which yields $0 = 55t - 2t^2$. This equation can be rewritten as $0 = t(55 - 2t)$. By the zero product property, it follows that $t = 0$ or $55 - 2t = 0$. Subtracting 55 from each side of the equation $55 - 2t = 0$ yields $-2t = -55$. Dividing each side of this equation by -2 yields $t = \frac{55}{2}$. Therefore, the zeros of function f are 0 and $\frac{55}{2}$. The value that's halfway between 0 and $\frac{55}{2}$ can be found by calculating the average of 0 and $\frac{55}{2}$, which is $\frac{0 + \frac{55}{2}}{2}$, or $\frac{55}{4}$. It follows that the maximum of function f occurs when $t = \frac{55}{4}$. Substituting $\frac{55}{4}$ for t in the equation defining function f yields $f(\frac{55}{4}) = 55(\frac{55}{4}) - 2(\frac{55}{4})^2$, which is equivalent to $f(\frac{55}{4}) = \frac{55^2}{4} - 2(\frac{55^2}{4^2})$. Multiplying $\frac{55^2}{4}$ by $\frac{4}{4}$ in this equation to get a common denominator yields $f(\frac{55}{4}) = 4(\frac{55^2}{4^2}) - 2(\frac{55^2}{4^2})$, or $f(\frac{55}{4}) = 2(\frac{55^2}{4^2})$, which is equivalent to $f(\frac{55}{4}) = 2(\frac{55}{4})^2$. Thus, the maximum value of $f(t)$ is $2(\frac{55}{4})^2$. Since the equation defining $g(t)$ is $g(t) = f(t) + 3$, the maximum value of $g(t)$ is 3 greater than the maximum value of $f(t)$. It follows that the maximum value of $g(t)$ is $3 + 2(\frac{55}{4})^2$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 42141e28

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 42141e28

$$f(x) = (x - 44)(x - 46)$$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

- A. 46
- B. 45
- C. 44
- D. -1

ID: 42141e28 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that $f(x) = (x - 44)(x - 46)$, which can be rewritten as $f(x) = x^2 - 90x + 2,024$. Since the coefficient of the x^2 -term is positive, the graph of $y = f(x)$ in the xy -plane opens upward and reaches its minimum value at its vertex. For an equation in the form $f(x) = ax^2 + bx + c$, where a , b , and c are constants, the x -coordinate of the vertex is $-\frac{b}{2a}$. For the equation $f(x) = x^2 - 90x + 2,024$, $a = 1$, $b = -90$, and $c = 2,024$. It follows that the x -coordinate of the vertex is $-\frac{(-90)}{2(1)}$, or 45. Therefore, $f(x)$ reaches its minimum when the value of x is 45.

Choice A is incorrect. This is one of the x -coordinates of the x -intercepts of the graph of $y = f(x)$ in the xy -plane.

Choice C is incorrect. This is one of the x -coordinates of the x -intercepts of the graph of $y = f(x)$ in the xy -plane.

Choice D is incorrect. This is the y -coordinate of the vertex of the graph of $y = f(x)$ in the xy -plane.

Question Difficulty: Hard

Question ID 92f8fab6

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 92f8fab6

The functions f and g are defined by the given equations, where $x \geq 0$. Which of the following equations displays, as a constant or coefficient, the maximum value of the function it defines, where $x \geq 0$?

I. $f(x) = 18(1.25)^x + 41$

II. $g(x) = 9(0.73)^x$

- A. I only
- B. II only
- C. I and II
- D. Neither I nor II

ID: 92f8fab6 Answer

Correct Answer: B

Rationale

Choice B is correct. For the function f , since the base of the exponent, 1.25 , is greater than 1 , the value of $(1.25)^x$ increases as x increases. Therefore, the value of $18(1.25)^x$ and the value of $18(1.25)^x + 41$ also increase as x increases. Since f is therefore an increasing function where $x \geq 0$, the function f has no maximum value. For the function g , since the base of the exponent, 0.73 , is less than 1 , the value of $(0.73)^x$ decreases as x increases. Therefore, the value of $9(0.73)^x$ also decreases as x increases. It follows that the maximum value of $g(x)$ for $x \geq 0$ occurs when $x = 0$. Substituting 0 for x in the function g yields $g(0) = 9(0.73)^0$, which is equivalent to $g(0) = 9(1)$, or $g(0) = 9$. Therefore, the maximum value of $g(x)$ for $x \geq 0$ is 9 , which appears as a coefficient in equation II. So, of the two equations given, only II displays, as a constant or coefficient, the maximum value of the function it defines, where $x \geq 0$.

- Choice A is incorrect and may result from conceptual or calculation errors.
- Choice C is incorrect and may result from conceptual or calculation errors.
- Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 032ab67d

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 032ab67d

$$f(t) = 8,000(0.65)^t$$

The given function f models the number of coupons a company sent to their customers at the end of each year, where t represents the number of years since the end of 1998, and $0 \leq t \leq 5$. If $y = f(t)$ is graphed in the ty -plane, which of the following is the best interpretation of the y -intercept of the graph in this context?

- A. The minimum estimated number of coupons the company sent to their customers during the 5 years was 1,428.
- B. The minimum estimated number of coupons the company sent to their customers during the 5 years was 8,000.
- C. The estimated number of coupons the company sent to their customers at the end of 1998 was 1,428.
- D. The estimated number of coupons the company sent to their customers at the end of 1998 was 8,000.

ID: 032ab67d Answer

Correct Answer: D

Rationale

Choice D is correct. The y -intercept of a graph in the ty -plane is the point where $t = 0$. For the given function f , the y -intercept of the graph of $y = f(t)$ in the ty -plane can be found by substituting 0 for t in the equation $y = 8,000(0.65)^t$, which gives $y = 8,000(0.65)^0$. This is equivalent to $y = 8,000(1)$, or $y = 8,000$. Therefore, the y -intercept of the graph of $y = f(t)$ is $(0, 8,000)$. It's given that the function f models the number of coupons a company sent to their customers at the end of each year. Therefore, $f(t)$ represents the estimated number of coupons the company sent to their customers at the end of each year. It's also given that t represents the number of years since the end of 1998. Therefore, $t = 0$ represents 0 years since the end of 1998, or the end of 1998. Thus, the best interpretation of the y -intercept of the graph of $y = f(t)$ is that the estimated number of coupons the company sent to their customers at the end of 1998 was 8,000.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 7b9287d0

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 7b9287d0

The function $f(w) = 6w^2$ gives the area of a rectangle, **in square feet (ft^2)**, if its width is w **ft** and its length is **6** times its width. Which of the following is the best interpretation of $f(14) = 1,176$?

- A. If the width of the rectangle is **14 ft**, then the area of the rectangle is **1,176 ft^2** .
- B. If the width of the rectangle is **14 ft**, then the length of the rectangle is **1,176 ft**.
- C. If the width of the rectangle is **1,176 ft**, then the length of the rectangle is **14 ft**.
- D. If the width of the rectangle is **1,176 ft**, then the area of the rectangle is **14 ft^2** .

ID: 7b9287d0 Answer

Correct Answer: A

Rationale

Choice A is correct. The function f gives the area of the rectangle, in ft^2 , if its width is w **ft**. Since the value of $f(14)$ is the value of $f(w)$ if $w = 14$, it follows that $f(14) = 1,176$ means that $f(w)$ is 1,176 if $w = 14$. In the given context, this means that if the width of the rectangle is **14 ft**, then the area of the rectangle is **1,176 ft^2** .

Choice B is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from interpreting $f(w)$ as the width, in **ft**, of the rectangle if its area is w ft^2 , rather than as the area, in ft^2 , of the rectangle if its width is w **ft**.

Question Difficulty: Hard

Question ID 90f01cb4

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 90f01cb4

The function f is defined by $f(x) = 5\left(\frac{1}{4} - x\right)^2 + \frac{11}{4}$. What is the value of $f\left(\frac{1}{4}\right)$?

ID: 90f01cb4 Answer

Correct Answer: 11/4, 2.75

Rationale

The correct answer is $\frac{11}{4}$. It's given that the function f is defined by $f(x) = 5\left(\frac{1}{4} - x\right)^2 + \frac{11}{4}$. Substituting $\frac{1}{4}$ for x in this equation yields $f\left(\frac{1}{4}\right) = 5\left(\frac{1}{4} - \frac{1}{4}\right)^2 + \frac{11}{4}$, which is equivalent $f\left(\frac{1}{4}\right) = 5(0)^2 + \frac{11}{4}$, or $f\left(\frac{1}{4}\right) = \frac{11}{4}$. Therefore, the value of $f\left(\frac{1}{4}\right)$ is $\frac{11}{4}$. Note that 11/4 or 2.75 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID bbeab5b6

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: bbeab5b6

The function f is defined by $f(x) = a^x + b$, where a and b are constants and $a > 0$. In the xy -plane, the graph of $y = f(x)$ has a y -intercept at $(0, -25)$ and passes through the point $(2, 23)$. What is the value of $a + b$?

ID: bbeab5b6 Answer

Correct Answer: -19

Rationale

The correct answer is -19 . It's given that function f is defined by $f(x) = a^x + b$, where a and b are constants and $a > 0$. It's also given that the graph of $y = f(x)$ in the xy -plane has a y -intercept at $(0, -25)$ and passes through the point $(2, 23)$. Since the graph has a y -intercept at $(0, -25)$, $f(0) = -25$. Substituting 0 for x in the given equation yields $f(0) = a^0 + b$, or $f(0) = 1 + b$, and substituting -25 for $f(0)$ in this equation yields $-25 = 1 + b$. Subtracting 1 from each side of this equation yields $-26 = b$. Substituting -26 for b in the equation $f(x) = a^x + b$ yields $f(x) = a^x - 26$. Since the graph also passes through the point $(2, 23)$, $f(2) = 23$. Substituting 2 for x in the equation $f(x) = a^x - 26$ yields $f(2) = a^2 - 26$, and substituting 23 for $f(2)$ yields $23 = a^2 - 26$. Adding 26 to each side of this equation yields $49 = a^2$. Taking the square root of both sides of this equation yields $\pm 7 = a$. Since it's given that $a > 0$, the value of a is 7 . It follows that the value of $a + b$ is $7 - 26$, or -19 .

Question Difficulty: Hard

Question ID 720dc199

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 720dc199

The function f is defined by $f(x) = 270(0.1)^x$. What is the value of $f(0)$?

- A. 0
- B. 1
- C. 27
- D. 270

ID: 720dc199 Answer

Correct Answer: D

Rationale

Choice D is correct. The value of $f(0)$ is the value of $f(x)$ when $x = 0$. Substituting 0 for x in the given function yields $f(0) = 270(0.1)^0$, or $f(0) = 270(1)$, which is equivalent to $f(0) = 270$. Therefore, the value of $f(0)$ is 270.

Choice A is incorrect. This is the value of x , not $f(x)$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the value of $f(1)$, not $f(0)$.

Question Difficulty: Hard

Question ID bb0a06b2

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: bb0a06b2

The function f is defined by $f(x) = 4x^{-1}$. What is the value of $f(21)$?

- A. -84
- B. $\frac{1}{84}$
- C. $\frac{4}{21}$
- D. $\frac{21}{4}$

ID: bb0a06b2 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that function f is defined by the equation $f(x) = 4x^{-1}$. The value of $f(21)$ is the value of $f(x)$ when $x = 21$. Substituting 21 for x in the given equation yields $f(21) = 4(21)^{-1}$, which is equivalent to $f(21) = 4(\frac{1}{21})$, or $f(21) = \frac{4}{21}$.

Choice A is incorrect. This is the value of $f(21)$ when $f(x) = -4x$, rather than $f(x) = 4x^{-1}$.

Choice B is incorrect. This is the value of $f(21)$ when $f(x) = (4x)^{-1}$, rather than $f(x) = 4x^{-1}$.

Choice D is incorrect. This is the value of $f(21)$ when $f(x) = (4^{-1})x$, rather than $f(x) = 4x^{-1}$.

Question Difficulty: Hard

Question ID 3d3e5400

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 3d3e5400

$q(x) = 32(2^x)$

Which table gives three values of x and their corresponding values of $q(x)$ for function q ?

A.

x	-1	0	1
$q(x)$	-64	0	64

B.

x	-1	0	1
$q(x)$	$\frac{1}{16}$	2	64

C.

x	-1	0	1
$q(x)$	$\frac{1}{16}$	32	64

D.

x	-1	0	1
$q(x)$	16	32	64

ID: 3d3e5400 Answer

Correct Answer: D

Rationale

Choice D is correct. Substituting -1 for x in the given function yields $q(-1) = 32(2)^{-1}$, which is equivalent to $q(-1) = 32(\frac{1}{2})$, or $q(-1) = 16$. Therefore, when $x = -1$, the corresponding value of $q(x)$ for function q is 16 . Substituting 0 for x in the given function yields $q(0) = 32(2)^0$, which is equivalent to $q(0) = 32(1)$, or $q(0) = 32$. Therefore, when $x = 0$, the corresponding value of $q(x)$ for function q is 32 . Substituting 1 for x in the given function yields $q(1) = 32(2)^1$, which is equivalent to $q(1) = 32(2)$, or $q(1) = 64$. Therefore, when $x = 1$, the corresponding value of $q(x)$ for function q is 64 . Of the choices given, only the table in choice D gives these three values of x and their corresponding values of $q(x)$ for function q .

Choice A is incorrect. This table gives three values of x and their corresponding values of $q(x)$ for the function $q(x) = 32(2x)$.

Choice B is incorrect. This table gives three values of x and their corresponding values of $q(x)$ for the function $q(x) = 2(32)^x$.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 8df14879

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 8df14879

$$f(x) = (1.84)^{\frac{x}{4}}$$

The function f is defined by the given equation. The equation can be rewritten as $f(x) = \left(1 + \frac{p}{100}\right)^x$, where p is a constant. Which of the following is closest to the value of p ?

- A. 16
- B. 21
- C. 46
- D. 96

ID: 8df14879 Answer

Correct Answer: A

Rationale

Choice A is correct. The equation $f(x) = (1.84)^{\frac{x}{4}}$ can be rewritten as $f(x) = (1.84)^{\left(\frac{1}{4}\right)(x)}$, which is equivalent to $f(x) = \left(1.84^{\frac{1}{4}}\right)^x$, or approximately $f(x) = (1.16467)^x$. Since it's given that $f(x) = (1.84)^{\frac{x}{4}}$ can be rewritten as $f(x) = \left(1 + \frac{p}{100}\right)^x$, where p is a constant, it follows that $1 + \frac{p}{100}$ is approximately equal to **1.16467**. Therefore, $\frac{p}{100}$ is approximately equal to **0.16467**. It follows that the value of p is approximately equal to **16.467**. Of the given choices, **16** is closest to the value of p .

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

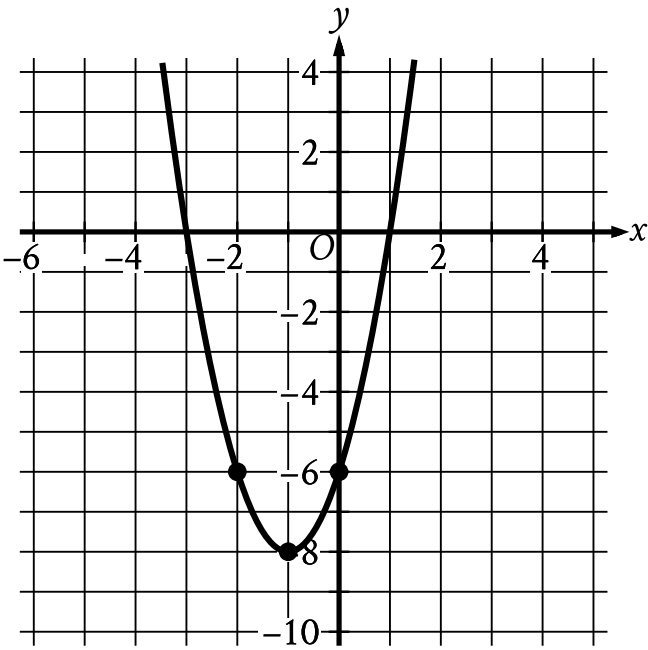
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 0a8ef03b

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 0a8ef03b



The graph of $y = 2x^2 + bx + c$ is shown, where b and c are constants. What is the value of bc ?

ID: 0a8ef03b Answer

Correct Answer: -24

Rationale

The correct answer is -24 . Since the graph passes through the point $(0, -6)$, it follows that when the value of x is 0 , the value of y is -6 . Substituting 0 for x and -6 for y in the given equation yields $-6 = 2(0)^2 + b(0) + c$, or $-6 = c$. Therefore, the value of c is -6 . Substituting -6 for c in the given equation yields $y = 2x^2 + bx - 6$. Since the graph passes through the point $(-1, -8)$, it follows that when the value of x is -1 , the value of y is -8 . Substituting -1 for x and -8 for y in the equation $y = 2x^2 + bx - 6$ yields $-8 = 2(-1)^2 + b(-1) - 6$, or $-8 = 2 - b - 6$, which is equivalent to $-8 = -4 - b$. Adding 4 to each side of this equation yields $-4 = -b$. Dividing each side of this equation by -1 yields $4 = b$. Since the value of b is 4 and the value of c is -6 , it follows that the value of bc is $(4)(-6)$, or -24 .

Alternate approach: The given equation represents a parabola in the xy -plane with a vertex at $(-1, -8)$. Therefore, the given equation, $y = 2x^2 + bx + c$, which is written in standard form, can be written in vertex form, $y = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola and a is the value of the coefficient on the x^2 term when the equation is written in standard form. It follows that $a = 2$. Substituting 2 for a , -1 for h , and -8 for k in this equation yields $y = 2(x - (-1))^2 + (-8)$, or $y = 2(x + 1)^2 - 8$. Squaring the binomial on the right-hand side of this equation yields $y = 2(x^2 + 2x + 1) - 8$. Multiplying each term inside the parentheses on the right-hand side of this equation by 2 yields $y = 2x^2 + 4x + 2 - 8$, which is equivalent to $y = 2x^2 + 4x - 6$. From the given equation $y = 2x^2 + bx + c$, it follows that the value of b is 4 and the value of c is -6 . Therefore, the value of bc is $(4)(-6)$, or -24 .

Question Difficulty: Hard

Question ID 39250844

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 39250844

$$y = 2(x - d)(x + d)(x + g)(x - d)$$

In the given equation, d and g are unique positive constants. When the equation is graphed in the xy -plane, how many distinct x -intercepts does the graph have?

- A. 4
- B. 3
- C. 2
- D. 1

ID: 39250844 Answer

Correct Answer: B

Rationale

Choice B is correct. An x -intercept of a graph in the xy -plane is a point on the graph where the value of y is 0 . Substituting 0 for y in the given equation yields $0 = 2(x - d)(x + d)(x + g)(x - d)$. By the zero product property, the solutions to this equation are $x = d$, $x = -d$, $x = -g$, and $x = d$. However, $x = d$ and $x = d$ are identical. It's given that d and g are unique positive constants. It follows that the equation $0 = 2(x - d)(x + d)(x + g)(x - d)$ has 3 unique solutions: $x = d$, $x = -d$, and $x = -g$. Thus, the graph of the given equation has 3 distinct x -intercepts.

Choice A is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 447489e5

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 447489e5

Time (years)	Total amount (dollars)
0	604.00
1	606.42
2	608.84

Rosa opened a savings account at a bank. The table shows the exponential relationship between the time t , in years, since Rosa opened the account and the total amount n , in dollars, in the account. If Rosa made no additional deposits or withdrawals, which of the following equations best represents the relationship between t and n ?

- A. $n = 604(1 + 0.004)^t$
- B. $n = 604(1 + 0.004)^t$
- C. $n = 604(1 + 0.004)^t$
- D. $n = 0.004(1 + 604)^t$

ID: 447489e5 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the relationship between t and n is exponential. The table shows that the value of n increases as the value of t increases. Therefore, the relationship between t and n can be represented by an increasing exponential equation of the form $n = a(1 + b)^t$, where a and b are positive constants. The table shows that when $t = 0$, $n = 604$. Substituting 0 for t and 604 for n in the equation $n = a(1 + b)^t$ yields $604 = a(1 + b)^0$, which is equivalent to $604 = a(1)$, or $604 = a$. Substituting 604 for a in the equation $n = a(1 + b)^t$ yields $n = 604(1 + b)^t$. The table also shows that when $t = 1$, $n = 606.42$. Substituting 1 for t and 606.42 for n in the equation $n = 604(1 + b)^t$ yields $606.42 = 604(1 + b)^1$, or $606.42 = 604(1 + b)$. Dividing both sides of this equation by 604 yields approximately $1.004 = 1 + b$. Subtracting 1 from both sides of this equation yields that the value of b is approximately 0.004. Substituting 0.004 for b in the equation $n = 604(1 + b)^t$ yields $n = 604(1 + 0.004)^t$. Therefore, of the choices, choice C best represents the relationship between t and n .

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 8aabeb91

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 8aabeb91

The function $f(t) = 40,000(2)^{\frac{t}{790}}$ gives the number of bacteria in a population t minutes after an initial observation. How much time, in minutes, does it take for the number of bacteria in the population to double?

- A. 2
- B. 790
- C. 1,580
- D. 40,000

ID: 8aabeb91 Answer

Correct Answer: B

Rationale

Choice B is correct. It’s given that t minutes after an initial observation, the number of bacteria in a population is $40,000(2)^{\frac{t}{790}}$. This expression consists of the initial number of bacteria, **40,000**, multiplied by the expression $2^{\frac{t}{790}}$. The time, in minutes, it takes for the number of bacteria to double is the increase in the value of t that causes the expression $2^{\frac{t}{790}}$ to double. Since the base is **2**, the expression $2^{\frac{t}{790}}$ will double when the exponent increases by **1**. Since the exponent of this expression is $\frac{t}{790}$, the exponent will increase by **1** when t increases by **790**. Therefore, the time, in minutes, it takes for the number of bacteria in the population to double is **790**.

Choice A is incorrect. This is the base of the exponent, not the time it takes for the number of bacteria in the population to double.

Choice C is incorrect. This is the number of minutes it takes for the population to double twice.

Choice D is incorrect. This is the number of bacteria that are initially observed, not the time it takes for the number of bacteria in the population to double.

Question Difficulty: Hard

Question ID 2179e29c

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 2179e29c

$$f(x) = 3,000(0.75)^x$$

A conservation scientist implemented a program to reduce the population of a certain species in an area. The given function estimates this species' population x years after 2008, where $x \leq 8$. Which of the following is the best interpretation of 3,000 in this context?

- A. The estimated percent decrease in the population for this species and area every 8 years after 2008
- B. The estimated percent decrease in the population for this species and area each year after 2008
- C. The estimated population for this species and area 8 years after 2008
- D. The estimated initial population for this species and area in 2008

ID: 2179e29c Answer

Correct Answer: D

Rationale

Choice D is correct. Substituting 0 for x in the given equation yields $f(0) = 3,000(0.75)^0$, which is equivalent to $f(0) = 3,000(1)$, or $f(0) = 3,000$. It's given that the function estimates the species' population x years after 2008, so it follows that the estimated population of the species is 3,000 in 2008. Therefore, the best interpretation of 3,000 in this context is the estimated initial population for this species and area in 2008.

Choice A is incorrect and may result from conceptual errors.

Choice B is incorrect. The estimated percent decrease in the population for this species and area each year after 2008 is 25%, not 3,000.

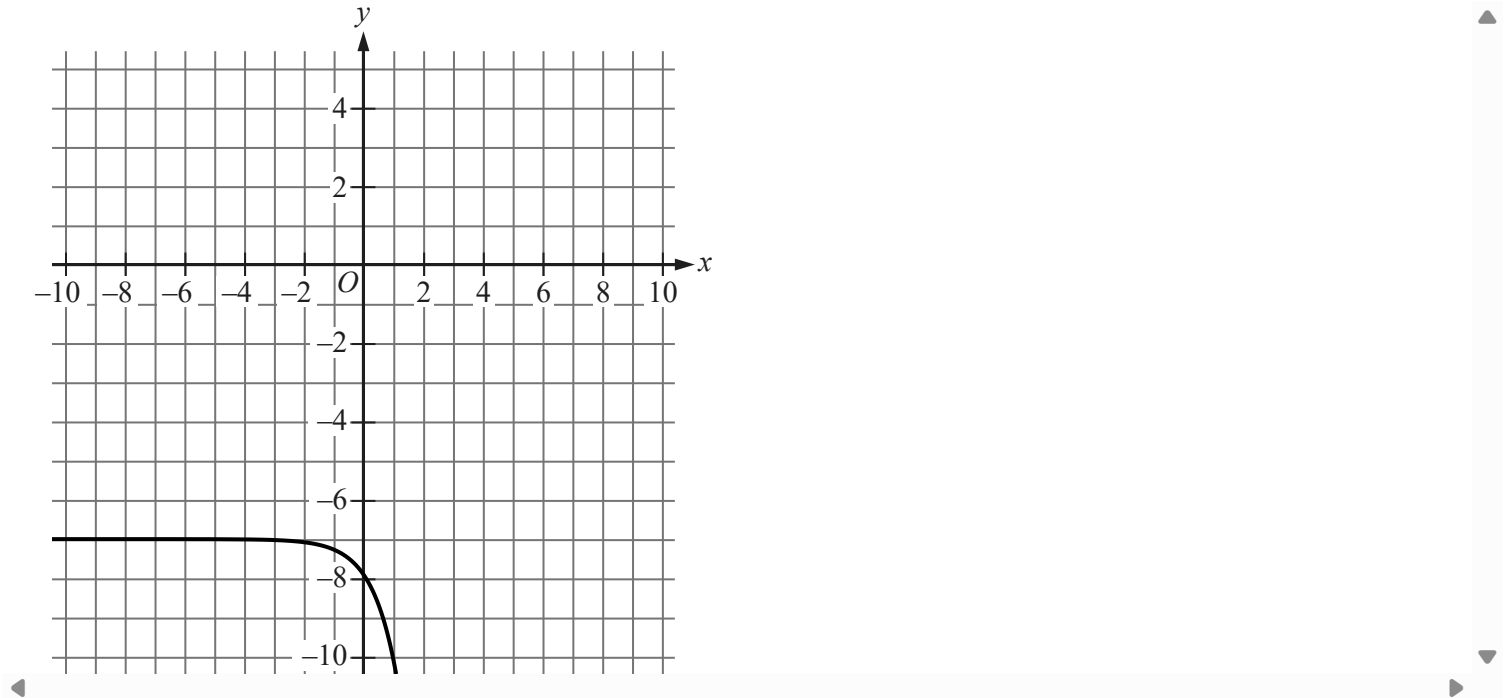
Choice C is incorrect. The estimated population for this species and area 8 years after 2008 is $3,000(0.75)^8$, or approximately 300, not 3,000.

Question Difficulty: Hard

Question ID d41063f7

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: d41063f7



The graph of $y = f(x)$ is shown, where $f(x) = ab^x + c$, and a , b , and c are constants. For how many values of x does $f(x) = 0$?

- A. Three
- B. Two
- C. One
- D. Zero

ID: d41063f7 Answer

Correct Answer: D

Rationale

Choice D is correct. Each point (x, y) on the graph of $y = f(x)$ in the xy -plane gives a value of x and its corresponding value of $f(x)$, or y . For any value of x for which $f(x) = 0$, there is a corresponding point on the graph of $y = f(x)$ with a y -coordinate of 0 . A point with a y -coordinate of 0 is a point where the graph intersects the x -axis. It's given that $f(x) = ab^x + c$, where a , b , and c are constants. In the xy -plane, the graph of an equation of this form will lie entirely either above or below the horizontal line defined by $y = c$. The part of the graph of $y = f(x)$ shown lies entirely below the horizontal line defined by $y = -7$, and thus the entire graph of $y = f(x)$ must lie below the line defined by $y = -7$. It follows that the graph of $y = f(x)$ will never intersect the x -axis. Therefore, there are zero values of x for which $f(x) = 0$.

Choice A is incorrect and may result from conceptual errors.

Choice B is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID cb504804

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: cb504804

A rectangle has an area of **155** square inches. The length of the rectangle is **4** inches less than **7** times the width of the rectangle. What is the width of the rectangle, in inches?

ID: cb504804 Answer

Correct Answer: 5

Rationale

The correct answer is **5**. Let x represent the width, in inches, of the rectangle. It's given that the length of the rectangle is **4** inches less than **7** times its width, or $7x - 4$ inches. The area of a rectangle is equal to its width multiplied by its length. Multiplying the width, x inches, by the length, $7x - 4$ inches, yields $x(7x - 4)$ square inches. It's given that the rectangle has an area of **155** square inches, so it follows that $x(7x - 4) = 155$, or $7x^2 - 4x = 155$. Subtracting **155** from both sides of this equation yields $7x^2 - 4x - 155 = 0$. Factoring the left-hand side of this equation yields $(7x + 31)(x - 5) = 0$. Applying the zero product property to this equation yields two solutions: $x = -\frac{31}{7}$ and $x = 5$. Since x is the rectangle's width, in inches, which must be positive, the value of x is **5**. Therefore, the width of the rectangle, in inches, is **5**.

Question Difficulty: Hard

Question ID e0703e6c

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: e0703e6c

The function f is defined by $f(x) = 8x^3 + 4$. What is the value of $f(2)$?

ID: e0703e6c Answer

Correct Answer: 68

Rationale

The correct answer is **68**. It's given that the function f is defined by $f(x) = 8x^3 + 4$. Substituting **2** for x in this equation yields $f(2) = 8(2)^3 + 4$, or $f(2) = 8(8) + 4$, which is equivalent to $f(2) = 68$. Therefore, the value of $f(2)$ is **68**.

Question Difficulty: Hard

Question ID c6c505f3

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: c6c505f3

The function f is defined by $f(x) = ax^2 + bx + c$, where a , b , and c are constants. The graph of $y = f(x)$ in the xy -plane passes through the points $(7, 0)$ and $(-3, 0)$. If a is an integer greater than 1 , which of the following could be the value of $a + b$?

- A. -6
- B. -3
- C. 4
- D. 5

ID: c6c505f3 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the graph of $y = f(x)$ in the xy -plane passes through the points $(7, 0)$ and $(-3, 0)$. It follows that when the value of x is either 7 or -3 , the value of $f(x)$ is 0 . It's also given that the function f is defined by $f(x) = ax^2 + bx + c$, where a , b , and c are constants. It follows that the function f is a quadratic function and, therefore, may be written in factored form as $f(x) = a(x - u)(x - v)$, where the value of $f(x)$ is 0 when x is either u or v . Since the value of $f(x)$ is 0 when the value of x is either 7 or -3 , and the value of $f(x)$ is 0 when the value of x is either u or v , it follows that u and v are equal to 7 and -3 . Substituting 7 for u and -3 for v in the equation $f(x) = a(x - u)(x - v)$ yields $f(x) = a(x - 7)(x - (-3))$, or $f(x) = a(x - 7)(x + 3)$. Distributing the right-hand side of this equation yields $f(x) = a(x^2 - 7x + 3x - 21)$, or $f(x) = ax^2 - 4ax - 21a$. Since it's given that $f(x) = ax^2 + bx + c$, it follows that $b = -4a$. Adding a to each side of this equation yields $a + b = -3a$. Since $a + b = -3a$, if a is an integer, the value of $a + b$ must be a multiple of 3 . If a is an integer greater than 1 , it follows that $a \geq 2$. Therefore, $-3a \leq -3(2)$. It follows that the value of $a + b$ is less than or equal to $-3(2)$, or -6 . Of the given choices, only -6 is a multiple of 3 that's less than or equal to -6 .

Choice B is incorrect. This is the value of $a + b$ if a is equal to, not greater than, 1 .

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 06c49455

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 06c49455

$$y = 0.25x^2 - 7.5x + 90.25$$

The equation gives the estimated stock price y , in dollars, for a certain company x days after a new product launched, where $0 \leq x \leq 20$. Which statement is the best interpretation of $(x, y) = (1, 83)$ in this context?

- A. The company's estimated stock price increased \$83 every day after the new product launched.
- B. The company's estimated stock price increased \$1 every 83 days after the new product launched.
- C. 1 day after the new product launched, the company's estimated stock price is \$83.
- D. 83 days after the new product launched, the company's estimated stock price is \$1.

ID: 06c49455 Answer

Correct Answer: C

Rationale

Choice C is correct. In the given equation, x represents the number of days after a new product launched, where $0 \leq x \leq 20$, and y represents the estimated stock price, in dollars, for a certain company. Therefore, the best interpretation of $(x, y) = (1, 83)$ in this context is that 1 day after the new product launched, the company's estimated stock price is \$83.

Choice A is incorrect and may result from conceptual errors.

Choice B is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 21e50bbb

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 21e50bbb

$$f(t) = 500(0.5)^{\frac{t}{12}}$$

The function f models the intensity of an X-ray beam, in number of particles in the X-ray beam, t millimeters below the surface of a sample of iron. According to the model, what is the estimated number of particles in the X-ray beam when it is at the surface of the sample of iron?

- A. 500
- B. 12
- C. 5
- D. 2

ID: 21e50bbb Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the function f models the intensity of an X-ray beam, in number of particles in the X-ray beam, t millimeters below the surface of a sample of iron. When the X-ray beam is at the surface of the sample of iron, it is 0 millimeters below the surface, so the value of t is 0 . Substituting 0 for t in the function $f(t) = 500(0.5)^{\frac{t}{12}}$ yields $f(0) = 500(0.5)^{\frac{0}{12}}$. Since any positive number raised to the power of 0 is equal to 1 , it follows that $f(0) = 500(1)$, or $f(0) = 500$. Therefore, the estimated number of particles in the X-ray beam at the surface of the sample of iron is 500 .

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 217e44de

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 217e44de

$$m(t) = -0.0274\left(\frac{t}{7}\right)^2 + 7.3873\left(\frac{t}{7}\right) + 75.032$$

The function m gives the predicted body mass $m(t)$, in **kilograms (kg)**, of a certain animal t days after it was born in a wildlife reserve, where $t \leq 390$. Which of the following is the best interpretation of the statement “ $m(330)$ is approximately equal to **362**” in this context?

- A. The predicted body mass of the animal was approximately **330 kg 362** days after it was born.
- B. The predicted body mass of the animal was approximately **362 kg 330** days after it was born.
- C. The predicted body mass of the animal was approximately **362 kg $\frac{330}{7}$** days after it was born.
- D. The predicted body mass of the animal was approximately **$\frac{330}{7}$ kg 362** days after it was born.

ID: 217e44de Answer

Correct Answer: B

Rationale

Choice B is correct. In the statement " $m(330)$ is approximately equal to **362**," the input of the function, **330**, is the value of t , the elapsed time, in days, since the animal was born. The approximate value of the function, **362**, is the predicted body mass, in kilograms, of the animal after that time has elapsed. Therefore, the predicted body mass of the animal was approximately **362 kg 330** days after it was born.

Choice A is incorrect. This would be the best interpretation of the statement " $m(362)$ is approximately equal to **330**."

Choice C is incorrect. The number $\frac{330}{7}$ is the number of weeks, not the number of days, after the animal was born.

Choice D is incorrect. This would be the best interpretation of the statement " $m(362)$ is approximately equal to $\frac{330}{7}$."

Question Difficulty: Hard

Question ID 1ade75f8

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 1ade75f8

On April 1, there were 233 views of an advertisement posted on a website. Every 2 days after April 1, the number of views of the advertisement had increased by 70% of the number of views 2 days earlier. The function f gives the predicted number of views x days after April 1. Which equation defines f ?

- A. $f(x) = 233(0.70)^{\frac{x}{2}}$
- B. $f(x) = 233(0.70)^{2x}$
- C. $f(x) = 233(1.70)^{\frac{x}{2}}$
- D. $f(x) = 233(1.70)^{2x}$

ID: 1ade75f8 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that on April 1, there were 233 views of the advertisement. It's also given that every 2 days after April 1, the number of views of the advertisement had increased by 70% of the number of views 2 days earlier. This situation can be represented by an exponential function of the form $f(x) = a(1 + \frac{r}{100})^{\frac{x}{c}}$, where a is the number of views on April 1 and every c days after April 1, the number of views had increased by $r\%$ of the number of views c days earlier. It follows that $a = 233$, $r = 70$, and $c = 2$. Substituting 233 for a , 70 for r , and 2 for c in the equation $f(x) = a(1 + \frac{r}{100})^{\frac{x}{c}}$ yields $f(x) = 233(1 + \frac{70}{100})^{\frac{x}{2}}$, or $f(x) = 233(1.70)^{\frac{x}{2}}$.

Choice A is incorrect. This function gives the predicted number of views for an advertisement for which every 2 days, the number of views was 70%, rather than increased by 70%, of the number of views 2 days earlier.

Choice B is incorrect. This function gives the predicted number of views for an advertisement for which every $\frac{1}{2}$ days, the number of views was 70% of the number of views $\frac{1}{2}$ days earlier, rather than an advertisement for which every 2 days, the number of views had increased by 70% of the number of views 2 days earlier.

Choice D is incorrect. This function gives the predicted number of views for an advertisement for which every $\frac{1}{2}$ days, rather than every 2 days, the number of views had increased by 70% of the number of views $\frac{1}{2}$ days earlier, rather than 2 days earlier.

Question Difficulty: Hard

Question ID aba35b4f

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: aba35b4f

The function f gives the product of a number, x , and a number that is **91** more than x . Which equation defines f ?

- A. $f(x) = x^2 + x + 91$
- B. $f(x) = x^2 + 91$
- C. $f(x) = x^2 + 91x$
- D. $f(x) = x^2 + 91x + 91$

ID: aba35b4f Answer

Correct Answer: C

Rationale

Choice C is correct. It’s given that the function f gives the product of a number, x , and a number that is **91** more than x . A number that is **91** more than x can be represented by the expression $x + 91$. Therefore, f can be defined by the equation $f(x) = x(x + 91)$, or $f(x) = x^2 + 91x$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 8664387e

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 8664387e

x	$g(x)$
-1	25
0	1
1	$\frac{1}{25}$
2	$\frac{1}{625}$

For the exponential function g , the table shows four values of x and their corresponding values of $g(x)$. Which equation defines g ?

- A. $g(x) = -25^x$
- B. $g(x) = -\left(\frac{1}{25}\right)^x$
- C. $g(x) = 25^x$
- D. $g(x) = \left(\frac{1}{25}\right)^x$

ID: 8664387e Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that function g is exponential. Therefore, an equation defining g can be written in the form $g(x) = a(b)^x$, where a and b are constants. The table shows that when $x = 0$, $g(x) = 1$. Substituting 0 for x and 1 for $g(x)$ in the equation $g(x) = a(b)^x$ yields $1 = a(b)^0$, which is equivalent to $1 = a$. Substituting 1 for a in the equation $g(x) = a(b)^x$ yields $g(x) = (b)^x$. The table also shows that when $x = 1$, $g(x) = \frac{1}{25}$. Substituting 1 for x and $\frac{1}{25}$ for $g(x)$ in the equation $g(x) = (b)^x$ yields $\frac{1}{25} = (b)^1$, which is equivalent to $\frac{1}{25} = b$. Substituting $\frac{1}{25}$ for b in the equation $g(x) = (b)^x$ yields $g(x) = \left(\frac{1}{25}\right)^x$.

Choice A is incorrect. For this function, $g(1)$ is equal to -25 , not $\frac{1}{25}$.

Choice B is incorrect. For this function, $g(1)$ is equal to $-\frac{1}{25}$, not $\frac{1}{25}$.

Choice C is incorrect. For this function, $g(1)$ is equal to 25 , not $\frac{1}{25}$.

Question Difficulty: Hard

Question ID e3fa3e11

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: e3fa3e11

$$f(x) = x^5 + 9x + 17$$

For the given function f , the graph of $y = f(x)$ in the xy -plane passes through the point $(0, b)$, where b is a constant. What is the value of b ?

ID: e3fa3e11 Answer

Correct Answer: 17

Rationale

The correct answer is **17**. It's given that the graph of $y = f(x)$ in the xy -plane passes through the point $(0, b)$, where b is a constant. It follows that $f(0)$ equals b . Substituting **0** for x in the given function yields $f(0) = 0^5 + 9(0) + 17$, or $f(0) = 17$. Therefore, the value of b is **17**.

Question Difficulty: Hard

Question ID 8112237d

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 8112237d

A park ranger hung squirrel houses each in the shape of a right rectangular prism for fox squirrels. Each house has a height of **11** inches. The length of each house's base is x inches, which is **1** inch more than the width of the house's base. Which function V gives the volume of each house, in cubic inches, in terms of the length of the house's base?

- A. $V(x) = 11x(x - 1)$
- B. $V(x) = 11x(x + 1)$
- C. $V(x) = x(x + 11)(x - 1)$
- D. $V(x) = x(x + 11)(x + 1)$

ID: 8112237d Answer

Correct Answer: A

Rationale

Choice A is correct. The volume of a prism is equal to the area of its base times its height. It's given that the length of each house's base is x inches and that this length is **1** inch more than the width, in inches, of the house's base. It follows that the width, in inches, of the house's base is $x - 1$. The area of a rectangle is the product of its length and its width. Therefore, the area of the base of the house is $x(x - 1)$ square inches. It's given that the height of each house is **11** inches. Therefore, the function V that gives the volume of each house, in cubic inches, in terms of the length of the house's base x is $V(x) = x(x - 1)11$, or $V(x) = 11x(x - 1)$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID e6bfb764

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: e6bfb764

An egg is thrown from a rooftop. The equation $h = -4.9t^2 + 9t + 18$ represents this situation, where h is the height of the egg above the ground, in meters, t seconds after it is thrown. According to the equation, what is the height, in meters, from which the egg was thrown?

ID: e6bfb764 Answer

Correct Answer: 18

Rationale

The correct answer is **18**. It's given that an egg is thrown from a rooftop and that the equation $h = -4.9t^2 + 9t + 18$ represents this situation, where h is the height of the egg above the ground, in meters, t seconds after it is thrown. It follows that the height, in meters, from which the egg was thrown is the value of h when $t = 0$. Substituting **0** for t in the equation $h = -4.9t^2 + 9t + 18$ yields $h(0) = -4.9(0)^2 + 9(0) + 18$, or $h = 18$. Therefore, according to the equation, the height, in meters, from which the egg was thrown is **18**.

Question Difficulty: Hard

Question ID 6a736803

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 6a736803

Immanuel purchased a certain rare coin on January 1. The function $f(x) = 65(1.03)^x$, where $0 \leq x \leq 10$, gives the predicted value, in dollars, of the rare coin x years after Immanuel purchased it. What is the best interpretation of the statement “ $f(8)$ is approximately equal to 82” in this context?

- A. When the rare coin's predicted value is approximately 82 dollars, it is 8% greater than the predicted value, in dollars, on January 1 of the previous year.
- B. When the rare coin's predicted value is approximately 82 dollars, it is 8 times the predicted value, in dollars, on January 1 of the previous year.
- C. From the day Immanuel purchased the rare coin to 8 years after Immanuel purchased the coin, its predicted value increased by a total of approximately 82 dollars.
- D. 8 years after Immanuel purchased the rare coin, its predicted value is approximately 82 dollars.

ID: 6a736803 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the function $f(x) = 65(1.03)^x$ gives the predicted value, in dollars, of a certain rare coin x years after Immanuel purchased it. It follows that $f(x)$ represents the predicted value, in dollars, of the coin x years after Immanuel purchased it. Since the value of $f(8)$ is the value of $f(x)$ when $x = 8$, it follows that “ $f(8)$ is approximately equal to 82” means that $f(x)$ is approximately equal to 82 when $x = 8$. Therefore, the best interpretation of the statement “ $f(8)$ is approximately equal to 82” in this context is 8 years after Immanuel purchased the rare coin, its predicted value is approximately 82 dollars.

Choice A is incorrect and may result from conceptual errors.

Choice B is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 879a67f5

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 879a67f5

The function f is defined by $f(x) = (-8)(2)^x + 22$. What is the y -intercept of the graph of $y = f(x)$ in the xy -plane?

- A. $(0, 14)$
- B. $(0, 2)$
- C. $(0, 22)$
- D. $(0, -8)$

ID: 879a67f5 Answer

Correct Answer: A

Rationale

Choice A is correct. The y -intercept of the graph of $y = f(x)$ in the xy -plane occurs at the point on the graph where $x = 0$. In other words, when $x = 0$, the corresponding value of $f(x)$ is the y -coordinate of the y -intercept. Substituting 0 for x in the given equation yields $f(0) = (-8)(2)^0 + 22$, which is equivalent to $f(0) = (-8)(1) + 22$, or $f(0) = 14$. Thus, when $x = 0$, the corresponding value of $f(x)$ is 14 . Therefore, the y -intercept of the graph of $y = f(x)$ in the xy -plane is $(0, 14)$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This could be the y -intercept for $f(x) = (-8)(2)^x$, not $f(x) = (-8)(2)^x + 22$.

Question Difficulty: Hard

Question ID b57f93cd

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: b57f93cd

The function g is defined by $g(x) = \frac{|x|}{a} - 14$, where $a < 0$. What is the product of $g(15a)$ and $g(7a)$?

ID: b57f93cd Answer

Correct Answer: 609

Rationale

The correct answer is **609**. It's given that the function g is defined by $g(x) = \frac{|x|}{a} - 14$, where $a < 0$. Substituting $15a$ for x in function g yields $g(15a) = \frac{|15a|}{a} - 14$. This function can be rewritten as $g(15a) = \frac{15|a|}{a} - 14$, or $g(15a) = 15\left(\frac{|a|}{a}\right) - 14$. Since $a < 0$, it follows that $\frac{|a|}{a} = -1$. Substituting -1 for $\frac{|a|}{a}$ in $g(15a) = 15\left(\frac{|a|}{a}\right) - 14$ yields $g(15a) = 15(-1) - 14$, or $g(15a) = -29$. Similarly, substituting $7a$ for x in function g yields $g(7a) = \frac{|7a|}{a} - 14$. This function can be rewritten as $g(7a) = \frac{7|a|}{a} - 14$, or $g(7a) = 7\left(\frac{|a|}{a}\right) - 14$. Since $a < 0$, it again follows that $\frac{|a|}{a} = -1$. Substituting -1 for $\frac{|a|}{a}$ in $g(7a) = 7\left(\frac{|a|}{a}\right) - 14$ yields $g(7a) = 7(-1) - 14$, or $g(7a) = -21$. Therefore, $g(15a) = -29$ and $g(7a) = -21$. Thus, the product of $g(15a)$ and $g(7a)$ is $(-29)(-21)$, or **609**.

Question Difficulty: Hard

Question ID 64a1e19a

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 64a1e19a

$f(x) = 272(2)^x$

The function f is defined by the given equation. If $h(x) = f(x - 4)$, which of the following equations defines function h ?

- A. $h(x) = 17(2)^x$
- B. $h(x) = 68(2)^x$
- C. $h(x) = 272(16)^x$
- D. $h(x) = 272(8)^x$

ID: 64a1e19a Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that $f(x) = 272(2)^x$ and $h(x) = f(x - 4)$. Substituting $x - 4$ for x in $f(x) = 272(2)^x$ yields $f(x - 4) = 272(2)^{x-4}$. Substituting $h(x)$ for $f(x - 4)$ in this equation yields $h(x) = 272(2)^{x-4}$. Using the properties of exponents, the function $h(x) = 272(2)^{x-4}$ can be rewritten as $h(x) = \frac{272(2)^x}{2^4}$, which is equivalent to $h(x) = \frac{272(2)^x}{16}$, or $h(x) = 17(2)^x$. Therefore, of the given choices, an equation that defines function h is $h(x) = 17(2)^x$.

Choice B is incorrect. This equation defines function h if $h(x) = f(x - 2)$, not $h(x) = f(x - 4)$.

Choice C is incorrect. This equation defines function h if $h(x) = f(4x)$, not $h(x) = f(x - 4)$.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID bbef0453

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: bbef0453

$f(x) = ax^2 + 4x + c$

In the given quadratic function, a and c are constants. The graph of $y = f(x)$ in the xy -plane is a parabola that opens upward and has a vertex at the point (h, k) , where h and k are constants. If $k < 0$ and $f(-9) = f(3)$, which of the following must be true?

- I. $c < 0$
 - II. $a \geq 1$
- A. I only
- B. II only
- C. I and II
- D. Neither I nor II

ID: bbef0453 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the graph of $y = f(x)$ in the xy -plane is a parabola with vertex (h, k) . If $f(-9) = f(3)$, then for the graph of $y = f(x)$, the point with an x -coordinate of -9 and the point with an x -coordinate of 3 have the same y -coordinate. In the xy -plane, a parabola is a symmetric graph such that when two points have the same y -coordinate, these points are equidistant from the vertex, and the x -coordinate of the vertex is halfway between the x -coordinates of these two points. Therefore, for the graph of $y = f(x)$, the points with x -coordinates -9 and 3 are equidistant from the vertex, (h, k) , and h is halfway between -9 and 3 . The value that is halfway between -9 and 3 is $\frac{-9+3}{2}$, or -3 . Therefore, $h = -3$. The equation defining f can also be written in vertex form, $f(x) = a(x - h)^2 + k$. Substituting -3 for h in this equation yields $f(x) = a(x - (-3))^2 + k$, or $f(x) = a(x + 3)^2 + k$. This equation is equivalent to $f(x) = a(x^2 + 6x + 9) + k$, or $f(x) = ax^2 + 6ax + 9a + k$. Since $f(x) = ax^2 + 4x + c$, it follows that $6a = 4$ and $9a + k = c$. Dividing both sides of the equation $6a = 4$ by 6 yields $a = \frac{4}{6}$, or $a = \frac{2}{3}$. Since $\frac{2}{3} < 1$, it's not true that $a \geq 1$. Therefore, statement II isn't true. Substituting $\frac{2}{3}$ for a in the equation $9a + k = c$ yields $9(\frac{2}{3}) + k = c$, or $6 + k = c$. Subtracting 6 from both sides of this equation yields $k = c - 6$. If $k < 0$, then $c - 6 < 0$, or $c < 6$. Since c could be any value less than 6 , it's not necessarily true that $c < 0$. Therefore, statement I isn't necessarily true. Thus, neither I nor II must be true.

- Choice A is incorrect and may result from conceptual or calculation errors.
- Choice B is incorrect and may result from conceptual or calculation errors.
- Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 9537c92e

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 9537c92e

$$f(x) = x^2 - 48x + 2,304$$

What is the minimum value of the given function?

ID: 9537c92e Answer

Correct Answer: 1728

Rationale

The correct answer is **1,728**. The given function can be rewritten in the form $f(x) = a(x - h)^2 + k$, where a is a positive constant and the minimum value, k , of the function occurs when the value of x is h . By completing the square, $f(x) = x^2 - 48x + 2,304$ can be written as $f(x) = x^2 - 48x + \left(\frac{48}{2}\right)^2 + 2,304 - \left(\frac{48}{2}\right)^2$, or $f(x) = (x - 24)^2 + 1,728$. This equation is in the form $f(x) = a(x - h)^2 + k$, where $a = 1$, $h = 24$, and $k = 1,728$. Therefore, the minimum value of the given function is **1,728**.

Question Difficulty: Hard

Question ID 778c50a3

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 778c50a3

The function f is defined by $f(x) = a\sqrt{x + b}$, where a and b are constants. In the xy -plane, the graph of $y = f(x)$ passes through the point $(-24, 0)$, and $f(24) < 0$. Which of the following must be true?

- A. $f(0) = 24$
- B. $f(0) = -24$
- C. $a > b$
- D. $a < b$

ID: 778c50a3 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that $f(24) < 0$. Substituting 24 for $f(x)$ in the equation $f(x) = a\sqrt{x + b}$ yields $f(24) = a\sqrt{24 + b}$. Therefore, $a\sqrt{24 + b} < 0$. Since $\sqrt{24 + b}$ can't be negative, it follows that $a < 0$. It's also given that the graph of $y = f(x)$ passes through the point $(-24, 0)$. It follows that when $x = -24$, $f(x) = 0$. Substituting -24 for x and 0 for $f(x)$ in the equation $f(x) = a\sqrt{x + b}$ yields $0 = a\sqrt{-24 + b}$. By the zero product property, either $a = 0$ or $\sqrt{-24 + b} = 0$. Since $a < 0$, it follows that $\sqrt{-24 + b} = 0$. Squaring both sides of this equation yields $-24 + b = 0$. Adding 24 to both sides of this equation yields $b = 24$. Since $a < 0$ and b is 24 , it follows that $a < b$ must be true.

- Choice A is incorrect. The value of $f(0)$ is $a\sqrt{b}$, which must be negative.
- Choice B is incorrect. The value of $f(0)$ is $a\sqrt{b}$, which could be -24 , but doesn't have to be.
- Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 5342ef3e

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 5342ef3e

An auditorium has seats for 1,800 people. Tickets to attend a show at the auditorium currently cost \$4.00. For each \$1.00 increase to the ticket price, 100 fewer tickets will be sold. This situation can be modeled by the equation $y = -100x^2 + 1,400x + 7,200$, where x represents the increase in ticket price, in dollars, and y represents the revenue, in dollars, from ticket sales. If this equation is graphed in the xy -plane, at what value of x is the maximum of the graph?

- A. 4
- B. 7
- C. 14
- D. 18

ID: 5342ef3e Answer

Correct Answer: B

Rationale

Choice B is correct. It’s given that the situation can be modeled by the equation $y = -100x^2 + 1,400x + 7,200$, where x represents the increase in ticket price, in dollars, and y represents the revenue, in dollars, from ticket sales. Since the coefficient of the x^2 term is negative, the graph of this equation in the xy -plane opens downward and reaches its maximum value at its vertex. If a quadratic equation in the form $y = ax^2 + bx + c$, where a , b , and c are constants, is graphed in the xy -plane, the x -coordinate of the vertex is equal to $-\frac{b}{2a}$. For the equation $y = -100x^2 + 1,400x + 7,200$, $a = -100$, $b = 1,400$, and $c = 7,200$. It follows that the x -coordinate of the vertex is $-\frac{1,400}{2(-100)}$, or 7. Therefore, if the given equation is graphed in the xy -plane, the maximum of the graph occurs at an x -value of 7.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 1c9eaa21

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 1c9eaa21

$$f(x) = (x - 1)(x + 3)(x - 2)$$

In the xy -plane, when the graph of the function f , where $y = f(x)$, is shifted up 6 units, the resulting graph is defined by the function g . If the graph of $y = g(x)$ crosses through the point $(4, b)$, where b is a constant, what is the value of b ?

ID: 1c9eaa21 Answer

Correct Answer: 48

Rationale

The correct answer is 48. It's given that in the xy -plane, when the graph of the function f , where $y = f(x)$, is shifted up 6 units, the resulting graph is defined by the function g . Therefore, function g can be defined by the equation $g(x) = f(x) + 6$. It's given that $f(x) = (x - 1)(x + 3)(x - 2)$. Substituting $(x - 1)(x + 3)(x - 2)$ for $f(x)$ in the equation $g(x) = f(x) + 6$ yields $g(x) = (x - 1)(x + 3)(x - 2) + 6$. For the point $(4, b)$, the value of x is 4. Substituting 4 for x in the equation $g(x) = (x - 1)(x + 3)(x - 2) + 6$ yields $g(4) = (4 - 1)(4 + 3)(4 - 2) + 6$, or $g(4) = 48$. It follows that the graph of $y = g(x)$ crosses through the point $(4, 48)$. Therefore, the value of b is 48.

Question Difficulty: Hard

Question ID b35da809

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: b35da809

What is an x-coordinate of an x-intercept of the graph of $y = 3(x - 14)(x + 5)(x + 4)$ in the xy-plane?

ID: b35da809 Answer

Correct Answer: 14, -5, -4

Rationale

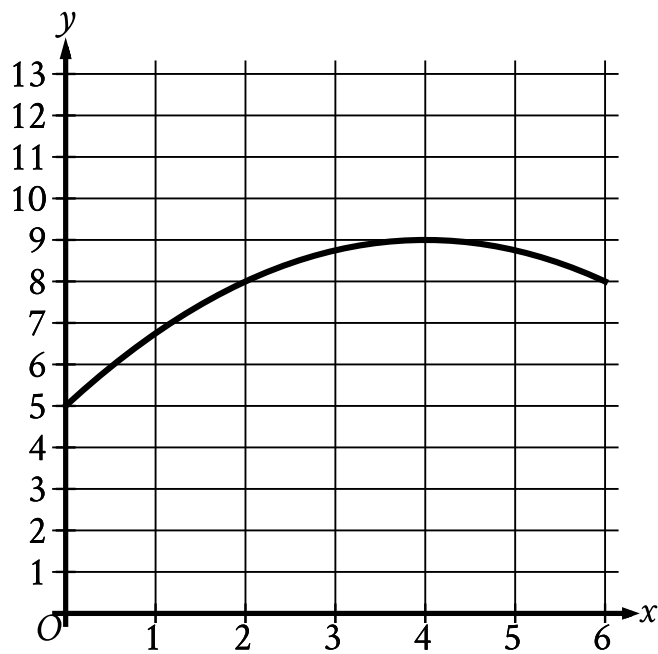
The correct answer is either **14**, **-5**, or **-4**. The x-intercepts of a graph in the xy-plane are the points at which the graph intersects the x-axis, or when the value of y is **0**. Substituting **0** for y in the given equation yields $0 = 3(x - 14)(x + 5)(x + 4)$. Dividing both sides of this equation by **3** yields $0 = (x - 14)(x + 5)(x + 4)$. Applying the zero product property to this equation yields three equations: $x - 14 = 0$, $x + 5 = 0$, and $x + 4 = 0$. Adding **14** to both sides of the equation $x - 14 = 0$ yields $x = 14$, subtracting **5** from both sides of the equation $x + 5 = 0$ yields $x = -5$, and subtracting **4** from both sides of the equation $x + 4 = 0$ yields $x = -4$. Therefore, the x-coordinates of the x-intercepts of the graph of the given equation are **14**, **-5**, and **-4**. Note that 14, -5, and -4 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 95614941

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 95614941



The graph models the number of active projects a company was working on x months after the end of November **2012**, where $0 \leq x \leq 6$. According to the model, what is the predicted number of active projects the company was working on at the end of November **2012**?

- A. 0
- B. 5
- C. 8
- D. 9

ID: 95614941 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the graph models the number of active projects a company was working on x months after the end of November **2012**. Therefore, the value of x that corresponds to the end of November **2012** is **0**. The point at which $x = 0$ is the y -intercept of the graph. It follows that the y -intercept of the graph shown is the point **(0, 5)**. Therefore, according to the model, the predicted number of active projects the company was working on at the end of November **2012** is **5**.

Choice A is incorrect. This is the value of x that corresponds to the end of November **2012**, not the predicted number of active projects the company was working on at the end of November **2012**.

Choice C is incorrect. This is the predicted number of active projects the company was working on **2** months after the end of November **2012**.

Choice D is incorrect. This is the predicted number of active projects the company was working on **4** months after the end of November **2012**.

Question Difficulty: Hard

Question ID 560d6aef

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 560d6aef

A physics class is planning an experiment about a toy rocket. The equation $y = -16(x - 5.6)^2 + 502$ gives the estimated height y , in feet, of the toy rocket x seconds after it is launched into the air. Which of the following is the best interpretation of the vertex of the graph of the equation in the xy -plane?

- A. This toy rocket reaches an estimated maximum height of **502** feet **16** seconds after it is launched into the air.
- B. This toy rocket reaches an estimated maximum height of **502** feet **5.6** seconds after it is launched into the air.
- C. This toy rocket reaches an estimated maximum height of **16** feet **502** seconds after it is launched into the air.
- D. This toy rocket reaches an estimated maximum height of **5.6** feet **502** seconds after it is launched into the air.

ID: 560d6aef Answer

Correct Answer: B

Rationale

Choice B is correct. The vertex of the graph of a quadratic equation is where it reaches its minimum or maximum value. When a quadratic equation is written in the form $y = a(x - h)^2 + k$, the vertex of the parabola represented by the equation is $(x, y) = (h, k)$. In the given equation $y = -16(x - 5.6)^2 + 502$, the value of h is **5.6** and the value of k is **502**. It follows that the vertex of the graph of this equation in the xy -plane is $(x, y) = (5.6, 502)$. Additionally, since $a = -16$ in the given equation, the graph of the quadratic equation opens down, and the vertex represents a maximum. It's given that the value of y represents the estimated height, in feet, of the toy rocket x seconds after it is launched into the air. Therefore, this toy rocket reaches an estimated maximum height of **502** feet **5.6** seconds after it is launched into the air.

Choice A is incorrect. The **16** in the equation is an indicator of how narrow the graph of the equation is rather than where it reaches its maximum.

Choice C is incorrect. The **16** in the equation is an indicator of how narrow the graph of the equation is rather than where it reaches its maximum.

Choice D is incorrect. This is an interpretation of the vertex of the graph of the equation $y = -16(x - 502)^2 + 5.6$, not of the equation $y = -16(x - 5.6)^2 + 502$.

Question Difficulty: Hard

Question ID e2d8deca

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: e2d8deca

A sample of a certain isotope takes **29** years to decay to half its original mass. The function $s(t) = 184(0.5)^{\frac{t}{29}}$ gives the approximate mass of this isotope, in grams, that remains t years after a **184**-gram sample starts to decay. Which statement is the best interpretation of $s(87) = 23$ in this context?

- A. Approximately **23** grams of the sample remains **87** years after the sample starts to decay.
- B. The mass of the sample has decreased by approximately **23** grams **87** years after the sample starts to decay.
- C. The mass of the sample has decreased by approximately **87** grams **23** years after the sample starts to decay.
- D. Approximately **87** grams of the sample remains **23** years after the sample starts to decay.

ID: e2d8deca Answer

Correct Answer: A

Rationale

Choice A is correct. In the given function, $s(t)$ represents the approximate mass, in grams, of the sample that remains t years after the sample starts to decay. It follows that the best interpretation of $s(87) = 23$ is that approximately **23** grams of the sample remains **87** years after the sample starts to decay.

Choice B is incorrect. The mass of the sample has decreased by approximately $184 - 23$, or **161**, grams, not **23** grams, **87** years after the sample starts to decay.

Choice C is incorrect. The mass of the sample has decreased by approximately **78** grams, not **87** grams, **23** years after the sample starts to decay.

Choice D is incorrect. This would be the best interpretation of $s(23) = 87$, not $s(87) = 23$.

Question Difficulty: Hard

Question ID d62dd40b

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: d62dd40b

The function g is defined by $g(x) = x(x - 2)(x + 6)^2$. The value of $g(7 - w)$ is 0, where w is a constant. What is the sum of all possible values of w ?

ID: d62dd40b Answer

Correct Answer: 25

Rationale

The correct answer is **25**. The value of $g(7 - w)$ is the value of $g(x)$ when $x = 7 - w$, where w is a constant. Substituting $7 - w$ for x in the given equation yields $g(7 - w) = (7 - w)(7 - w - 2)(7 - w + 6)^2$, which is equivalent to $g(7 - w) = (7 - w)(5 - w)(13 - w)^2$. It's given that the value of $g(7 - w)$ is 0. Substituting 0 for $g(7 - w)$ in the equation $g(7 - w) = (7 - w)(5 - w)(13 - w)^2$ yields $0 = (7 - w)(5 - w)(13 - w)^2$. Since the product of the three factors on the right-hand side of this equation is equal to 0, at least one of these three factors must be equal to 0. Therefore, the possible values of w can be found by setting each factor equal to 0. Setting the first factor equal to 0 yields $7 - w = 0$. Adding w to both sides of this equation yields $7 = w$. Therefore, 7 is one possible value of w . Setting the second factor equal to 0 yields $5 - w = 0$. Adding w to both sides of this equation yields $5 = w$. Therefore, 5 is a second possible value of w . Setting the third factor equal to 0 yields $(13 - w)^2 = 0$. Taking the square root of both sides of this equation yields $13 - w = 0$. Adding w to both sides of this equation yields $13 = w$. Therefore, 13 is a third possible value of w . Adding the three possible values of w yields $7 + 5 + 13$, or **25**. Therefore, the sum of all possible values of w is **25**.

Question Difficulty: Hard

Question ID 9e589c7d

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 9e589c7d

x	$g(x)$
-27	3
-9	0
21	5

The table shows three values of x and their corresponding values of $g(x)$, where $g(x) = \frac{f(x)}{x+3}$ and f is a linear function. What is the y-intercept of the graph of $y = f(x)$ in the xy -plane?

- A. $(0, 36)$
- B. $(0, 12)$
- C. $(0, 4)$
- D. $(0, -9)$

ID: 9e589c7d Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the table shows values of x and their corresponding values of $g(x)$, where $g(x) = \frac{f(x)}{x+3}$. It's also given that f is a linear function. It follows that an equation that defines f can be written in the form $f(x) = mx + b$, where m represents the slope and b represents the y-coordinate of the y-intercept $(0, b)$ of the graph of $y = f(x)$ in the xy -plane. The slope of the graph of $y = f(x)$ can be found using two points, (x_1, y_1) and (x_2, y_2) , that are on the graph of $y = f(x)$, and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Since the table shows values of x and their corresponding values of $g(x)$, substituting values of x and $g(x)$ in the equation $g(x) = \frac{f(x)}{x+3}$ can be used to define function f . Using the first pair of values from the table, $x = -27$ and $g(x) = 3$, yields $3 = \frac{f(-27)}{-27+3}$, or $3 = \frac{f(-27)}{-24}$. Multiplying each side of this equation by -24 yields $-72 = f(-27)$, so the point $(-27, -72)$ is on the graph of $y = f(x)$. Using the second pair of values from the table, $x = -9$ and $g(x) = 0$, yields $0 = \frac{f(-9)}{-9+3}$, or $0 = \frac{f(-9)}{-6}$. Multiplying each side of this equation by -6 yields $0 = f(-9)$, so the point $(-9, 0)$ is on the graph of $y = f(x)$. Substituting $(-27, -72)$ and $(-9, 0)$ for (x_1, y_1) and (x_2, y_2) , respectively, in the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ yields $m = \frac{0 - (-72)}{-9 - (-27)}$, or $m = 4$. Substituting 4 for m in the equation $f(x) = mx + b$ yields $f(x) = 4x + b$. Since $0 = f(-9)$, substituting -9 for x and 0 for $f(x)$ in the equation $f(x) = 4x + b$ yields $0 = 4(-9) + b$, or $0 = -36 + b$. Adding 36 to both sides of this equation yields $36 = b$. It follows that 36 is the y-coordinate of the y-intercept $(0, b)$ of the graph of $y = f(x)$. Therefore, the y-intercept of the graph of $y = f(x)$ is $(0, 36)$.

Choice B is incorrect. 12 is the y-coordinate of the y-intercept of the graph of $y = g(x)$.

Choice C is incorrect. 4 is the slope of the graph of $y = f(x)$.

Choice D is incorrect. -9 is the x -coordinate of the x -intercept of the graph of $y = f(x)$.

Question Difficulty: Hard

Question ID 1d689159

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 1d689159

$$h(t) = -16t^2 + b$$

The function h estimates an object’s height, in feet, above the ground t seconds after the object is dropped, where b is a constant. The function estimates that the object is **3,364** feet above the ground when it is dropped at $t = 0$. Approximately how many seconds after being dropped does the function estimate the object will hit the ground?

- A. **7.25**
- B. **14.50**
- C. **105.13**
- D. **210.25**

ID: 1d689159 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the function h estimates that the object is **3,364** feet above the ground when it's dropped at $t = 0$. Substituting **3,364** for $h(t)$ and **0** for t in the function h yields $3,364 = -16(0)^2 + b$, or $3,364 = b$. Substituting **3,364** for b in the function h yields $h(t) = -16t^2 + 3,364$. When the object hits the ground, its height will be **0** feet above the ground. Substituting **0** for $h(t)$ in $h(t) = -16t^2 + 3,364$ yields $0 = -16t^2 + 3,364$. Adding $16t^2$ to each side of this equation yields $16t^2 = 3,364$. Dividing each side of this equation by **16** yields $t^2 = 210.25$. Since the object will hit the ground at a positive number of seconds after it's dropped, the value of t can be found by taking the positive square root of each side of this equation, which yields $t = 14.50$. It follows that the function estimates the object will hit the ground approximately **14.50** seconds after being dropped.

Choice A is incorrect. The function estimates that **7.25** seconds after being dropped, the object's height will be $-16(7.25)^2 + 3,364$ feet, or **2,523** feet, above the ground.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID adfb10a0

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: adfb10a0

Time (years)	Total amount (dollars)
0	670.00
1	674.02
2	678.06

Sara opened a savings account at a bank. The table shows the exponential relationship between the time t , in years, since Sara opened the account and the total amount d , in dollars, in the account. If Sara made no additional deposits or withdrawals, which of the following equations best represents the relationship between t and d ?

- A. $d = 0.006$
- B. $d = 670$
- C. $d =$
- D. $d =$

ID: adfb10a0 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the relationship between t and d is exponential. The table shows that the value of d increases as the value of t increases. Therefore, the relationship between t and d can be represented by an increasing exponential equation of the form $d = a(1 + b)^t$, where a and b are positive constants. The table shows that when $t = 0$, $d = 670$. Substituting 0 for t and 670 for d in the equation $d = a(1 + b)^t$ yields $670 = a(1 + b)^0$, which is equivalent to $670 = a(1)$, or $670 = a$. Substituting 670 for a in the equation $d = a(1 + b)^t$ yields $d = 670(1 + b)^t$. The table also shows that when $t = 1$, $d = 674.02$. Substituting 1 for t and 674.02 for d in the equation $d = 670(1 + b)^t$ yields $674.02 = 670(1 + b)^1$, or $674.02 = 670(1 + b)$. Dividing both sides of this equation by 670 yields $1.006 = 1 + b$. Subtracting 1 from both sides of this equation yields $b = 0.006$. Substituting 0.006 for b in the equation $d = 670(1 + b)^t$ yields $d = 670(1 + 0.006)^t$. Therefore, of the choices, choice B best represents the relationship between t and d .

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 14e8f4fa

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 14e8f4fa

x	$h(x)$
0	1.23
2	1.54
4	1.94

The table shows the exponential relationship between the number of years, x , since Hana started training in pole vault, and the estimated height $h(x)$, in meters, of her best pole vault for that year. Which of the following functions best represents this relationship, where $x \leq 4$?

- A. $h(x) = 1.12(0.23)^x$
- B. $h(x) = 1.12(1.23)^x$
- C. $h(x) = 1.23(0.12)^x$
- D. $h(x) = 1.23(1.12)^x$

ID: 14e8f4fa Answer

Correct Answer: D

Rationale

Choice D is correct. The table shows an increasing exponential relationship between the number of years, x , since Hana started training in pole vault and the estimated height $h(x)$, in meters, of her best pole vault for that year. The relationship can be written as $h(x) = Ca^x$, where C and a are positive constants. It's given that when $x = 0$, $h(x) = 1.23$. Substituting 0 for x and 1.23 for $h(x)$ in $h(x) = Ca^x$ yields $1.23 = Ca^0$, or $1.23 = C$. Substituting 1.23 for C in $h(x) = Ca^x$ yields $h(x) = 1.23a^x$. It's also given that when $x = 2$, $h(x) = 1.54$. Substituting 2 for x and 1.54 for $h(x)$ in $h(x) = 1.23a^x$ yields $1.54 = 1.23a^2$. Dividing each side of this equation by 1.23 yields $\frac{1.54}{1.23} = \frac{1.23a^2}{1.23}$, or a^2 is approximately equal to 1.252. Since a is positive, a is approximately equal to $\sqrt{1.252}$, or 1.12. Substituting 1.12 for a in $h(x) = 1.23a^x$ yields $h(x) = 1.23(1.12)^x$.

Choice A is incorrect. When $x = 0$, the value of $h(x)$ in this function is equal to 1.12 rather than 1.23, and it is decreasing rather than increasing.

Choice B is incorrect. When $x = 0$, the value of $h(x)$ in this function is equal to 1.12 rather than 1.23.

Choice C is incorrect. This function is decreasing rather than increasing.

Question Difficulty: Hard

Question ID 4623a954

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 4623a954

A submersible device is used for ocean research. The function $g(x) = -\frac{1}{55}(x + 19)(x - 35)$ gives the depth below the surface of the ocean, in meters, of the submersible device x minutes after collecting a sample, where $x > 0$. How many minutes after collecting the sample did it take for the submersible device to reach the surface of the ocean?

ID: 4623a954 Answer

Correct Answer: 35

Rationale

The correct answer is **35**. It's given that the function $g(x) = -\frac{1}{55}(x + 19)(x - 35)$ gives the depth below the surface of the ocean, in meters, of the submersible device x minutes after collecting a sample, where $x > 0$. It follows that when the submersible device is at the surface of the ocean, the value of $g(x)$ is **0**. Substituting **0** for $g(x)$ in the equation $g(x) = -\frac{1}{55}(x + 19)(x - 35)$ yields $0 = -\frac{1}{55}(x + 19)(x - 35)$. Multiplying both sides of this equation by -55 yields $0 = (x + 19)(x - 35)$. Since a product of two factors is equal to **0** if and only if at least one of the factors is **0**, either $x + 19 = 0$ or $x - 35 = 0$. Subtracting **19** from both sides of the equation $x + 19 = 0$ yields $x = -19$. Adding **35** to both sides of the equation $x - 35 = 0$ yields $x = 35$. Since $x > 0$, **35** minutes after collecting the sample the submersible device reached the surface of the ocean.

Question Difficulty: Hard

Question ID 883e8024

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 883e8024

A function p estimates that there were **2,000** animals in a population in **1998**. Each year from **1998** to **2010**, the function estimates that the number of animals in this population increased by **3%** of the number of animals in the population the previous year. Which equation defines this function, where $p(x)$ is the estimated number of animals in the population x years after **1998**?

- A. $p(x) = 2,000(3)^x$
- B. $p(x) = 2,000(1.97)^x$
- C. $p(x) = 2,000(1.03)^x$
- D. $p(x) = 2,000(0.97)^x$

ID: 883e8024 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that a function p estimates that there were **2,000** animals in a population in **1998** and that each year from **1998** to **2010**, the number of animals in this population increased by **3%** of the number of animals in the population the previous year. It follows that this situation can be represented by the function $p(x) = a\left(1 + \frac{r}{100}\right)^x$, where $p(x)$ is the estimated number of animals in the population x years after **1998**, a is the estimated number of animals in the population in **1998**, and each year the estimated number of animals increased by $r\%$. Substituting **2,000** for a and **3** for r in this function yields $p(x) = 2,000\left(1 + \frac{3}{100}\right)^x$, or $p(x) = 2,000(1.03)^x$.

Choice A is incorrect. This function represents a population in which each year the number of animals increased by **200%**, not **3%**, of the number of animals in the population the previous year.

Choice B is incorrect. This function represents a population in which each year the number of animals increased by **97%**, not **3%**, of the number of animals in the population the previous year.

Choice D is incorrect. This function represents a population in which each year the number of animals decreased, rather than increased, by **3%** of the number of animals in the population the previous year.

Question Difficulty: Hard

Question ID d662c359

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: d662c359

The function g is defined by $g(x) = (x + 14)(t - x)$, where t is a constant. In the xy -plane, the graph of $y = g(x)$ passes through the point $(24, 0)$. What is the value of $g(0)$?

ID: d662c359 Answer

Correct Answer: 336

Rationale

The correct answer is **336**. By the zero product property, if $(x + 14)(t - x) = 0$, then $x + 14 = 0$, which gives $x = -14$, or $(t - x) = 0$, which gives $x = t$. Therefore, $g(x) = 0$ when $x = -14$ and when $x = t$. Since the graph of $y = g(x)$ passes through the point $(24, 0)$, it follows that $g(24) = 0$, so $t = 24$. Substituting **24** for t in the equation $g(x) = (x + 14)(t - x)$ yields $g(x) = (x + 14)(24 - x)$. The value of $g(0)$ can be calculated by substituting **0** for x in this equation, which yields $g(0) = (0 + 14)(24 - 0)$, or $g(0) = 336$.

Question Difficulty: Hard

Question ID 5ce507ae

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 5ce507ae

x	y
1	11
2	19
3	a

The table shows three values of x and their corresponding values of y for the equation $y = 4(2)^x + 3$. In the table, a is a constant. What is the value of a ?

- A. 67
- B. 35
- C. 32
- D. 27

ID: 5ce507ae Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the table shows three values of x and their corresponding values of y for the equation $y = 4(2)^x + 3$. It's also given that when $x = 3$ the corresponding value of y is a , and a is a constant. Substituting 3 for x and a for y in the given equation yields $a = 4(2)^3 + 3$, or $a = 35$. Therefore, the value of a is 35.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 0ed88ab6

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 0ed88ab6

The function f is defined by $f(x) = a^x + b$, where a and b are constants. In the xy -plane, the graph of $y = f(x)$ has an x -intercept at $(2, 0)$ and a y -intercept at $(0, -323)$. What is the value of b ?

ID: 0ed88ab6 Answer

Correct Answer: -324

Rationale

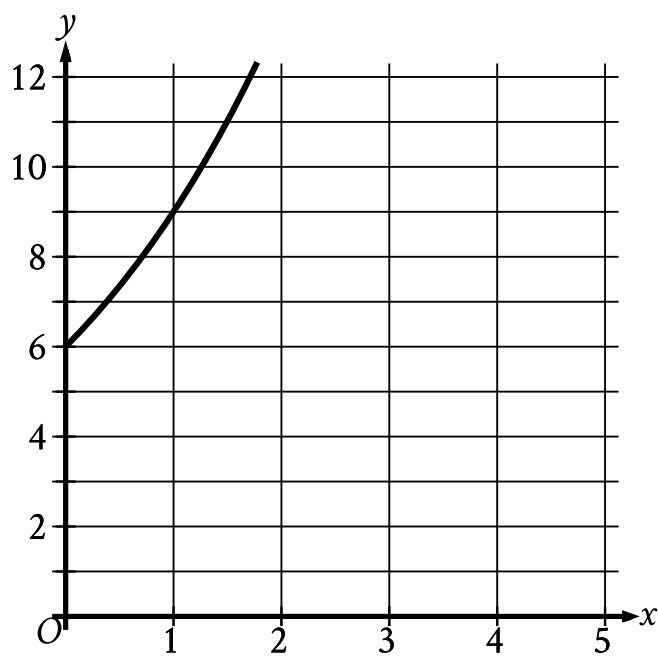
The correct answer is -324 . It's given that the function f is defined by $f(x) = a^x + b$, where a and b are constants. It's also given that the graph of $y = f(x)$ has a y -intercept at $(0, -323)$. It follows that $f(0) = -323$. Substituting 0 for x and -323 for $f(x)$ in $f(x) = a^x + b$ yields $-323 = a^0 + b$, or $-323 = 1 + b$. Subtracting 1 from each side of this equation yields $-324 = b$. Therefore, the value of b is -324 .

Question Difficulty: Hard

Question ID 23cc8869

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 23cc8869



The graph gives the estimated population y , in thousands, of a town x years since 2003, where $0 \leq x \leq 5$. Which of the following best describes the increase in the estimated population from $x = 0$ to $x = 1$?

- A. The estimated population at $x = 1$ is 0.5 times the estimated population at $x = 0$.
- B. The estimated population at $x = 1$ is 1.5 times the estimated population at $x = 0$.
- C. The estimated population at $x = 1$ is 2.5 times the estimated population at $x = 0$.
- D. The estimated population at $x = 1$ is 3.5 times the estimated population at $x = 0$.

ID: 23cc8869 Answer

Correct Answer: B

Rationale

Choice B is correct. On the graph shown, the y -axis represents estimated population, in thousands. The graph shows that when $x = 0$, the y -coordinate is 6. Therefore, the estimated population at $x = 0$ is 6 thousand. The graph also shows that when $x = 1$, the y -coordinate is 9. Therefore, the estimated population at $x = 1$ is 9 thousand. Dividing 9 thousand by 6 thousand yields 1.5; therefore, 9 thousand is 1.5 times 6 thousand. It follows that the estimated population at $x = 1$ is 1.5 times the estimated population at $x = 0$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question ID 16749ce5

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 16749ce5

x	$f(x)$
-1	10
0	14
1	20

For the quadratic function f , the table shows three values of x and their corresponding values of $f(x)$. Which equation defines f ?

- A. $f(x) = 3x^2 + 3x + 14$
- B. $f(x) = 5x^2 + x + 14$
- C. $f(x) = 9x^2 - x + 14$
- D. $f(x) = x^2 + 5x + 14$

ID: 16749ce5 Answer

Correct Answer: D

Rationale

Choice D is correct. The equation of a quadratic function can be written in the form $f(x) = a(x - h)^2 + k$, where a , h , and k are constants. It's given in the table that when $x = -1$, the corresponding value of $f(x)$ is 10 . Substituting -1 for x and 10 for $f(x)$ in the equation $f(x) = a(x - h)^2 + k$ gives $10 = a(-1 - h)^2 + k$, which is equivalent to $10 = a(1 + 2h + h^2) + k$, or $10 = a + 2ah + ah^2 + k$. It's given in the table that when $x = 0$, the corresponding value of $f(x)$ is 14 . Substituting 0 for x and 14 for $f(x)$ in the equation $f(x) = a(x - h)^2 + k$ gives $14 = a(0 - h)^2 + k$, or $14 = ah^2 + k$. It's given in the table that when $x = 1$, the corresponding value of $f(x)$ is 20 . Substituting 1 for x and 20 for $f(x)$ in the equation $f(x) = a(x - h)^2 + k$ gives $20 = a(1 - h)^2 + k$, which is equivalent to $20 = a(1 - 2h + h^2) + k$, or $20 = a - 2ah + ah^2 + k$. Adding $20 = a - 2ah + ah^2 + k$ to the equation $10 = a + 2ah + ah^2 + k$ gives $30 = 2a + 2ah^2 + 2k$. Dividing both sides of this equation by 2 gives $15 = a + ah^2 + k$. Since $14 = ah^2 + k$, substituting 14 for $ah^2 + k$ into the equation $15 = a + ah^2 + k$ gives $15 = a + 14$. Subtracting 14 from both sides of this equation gives $a = 1$. Substituting 1 for a in the equations $14 = ah^2 + k$ and $20 = ah^2 - 2ah + a + k$ gives $14 = h^2 + k$ and $20 = 1 - 2h + h^2 + k$, respectively. Since $14 = h^2 + k$, substituting 14 for $h^2 + k$ in the equation $20 = 1 - 2h + h^2 + k$ gives $20 = 1 - 2h + 14$, or $20 = 15 - 2h$. Subtracting 15 from both sides of this equation gives $5 = -2h$. Dividing both sides of this equation by -2 gives $-\frac{5}{2} = h$. Substituting $-\frac{5}{2}$ for h into the equation $14 = h^2 + k$ gives $14 = (-\frac{5}{2})^2 + k$, or $14 = \frac{25}{4} + k$. Subtracting $\frac{25}{4}$ from both sides of this equation gives $\frac{31}{4} = k$. Substituting 1 for a , $-\frac{5}{2}$ for h , and $\frac{31}{4}$ for k in the equation $f(x) = a(x - h)^2 + k$ gives $f(x) = (x + \frac{5}{2})^2 + \frac{31}{4}$, which is equivalent to $f(x) = x^2 + 5x + \frac{25}{4} + \frac{31}{4}$, or $f(x) = x^2 + 5x + 14$. Therefore, $f(x) = x^2 + 5x + 14$ defines f .

Choice A is incorrect. If $f(x) = 3x^2 + 3x + 14$, then when $x = -1$, the corresponding value of $f(x)$ is 14 , not 10 .

Choice B is incorrect. If $f(x) = 5x^2 + x + 14$, then when $x = -1$, the corresponding value of $f(x)$ is 18 , not 10 .

Choice C is incorrect. If $f(x) = 9x^2 - x + 14$, then when $x = -1$, the corresponding value of $f(x)$ is **24**, not **10**, and when $x = 1$, the corresponding value of $f(x)$ is **22**, not **20**.

Question Difficulty: Hard

Question ID f9bbe9d9

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: f9bbe9d9

The function $f(x) = 206(1.034)^x$ models the value, in dollars, of a certain bank account by the end of each year from 1957 through 1972, where x is the number of years after 1957. Which of the following is the best interpretation of “ $f(5)$ is approximately equal to 243” in this context?

- A. The value of the bank account is estimated to be approximately 5 dollars greater in 1962 than in 1957.
- B. The value of the bank account is estimated to be approximately 243 dollars in 1962.
- C. The value, in dollars, of the bank account is estimated to be approximately 5 times greater in 1962 than in 1957.
- D. The value of the bank account is estimated to increase by approximately 243 dollars every 5 years between 1957 and 1972.

ID: f9bbe9d9 Answer

Correct Answer: B

Rationale

Choice B is correct. It’s given that the function $f(x) = 206(1.034)^x$ models the value, in dollars, of a certain bank account by the end of each year from 1957 through 1972, where x is the number of years after 1957. It follows that $f(x)$ represents the estimated value, in dollars, of the bank account x years after 1957. Since the value of $f(5)$ is the value of $f(x)$ when $x = 5$, it follows that “ $f(5)$ is approximately equal to 243” means that $f(x)$ is approximately equal to 243 when $x = 5$. In the given context, this means that the value of the bank account is estimated to be approximately 243 dollars 5 years after 1957. Therefore, the best interpretation of the statement “ $f(5)$ is approximately equal to 243” in this context is the value of the bank account is estimated to be approximately 243 dollars in 1962.

Choice A is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 0d567847

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 0d567847

The area of a triangle is **270** square centimeters. The length of the base of the triangle is **12** centimeters greater than the height of the triangle. What is the height, in centimeters, of the triangle?

- A. 15
- B. 18
- C. 30
- D. 36

ID: 0d567847 Answer

Correct Answer: B

Rationale

Choice B is correct. The area, **A**, of a triangle is given by the formula $A = \frac{1}{2}bh$, where **b** represents the length of the base of the triangle and **h** represents its height. It's given that the area of a triangle is **270** square centimeters and that the length of the base of this triangle is **12** centimeters greater than the height of the triangle. Let **x** represent the height, in centimeters, of the triangle. It follows that the length of the base of the triangle can be expressed as **x + 12**. Substituting **270** for **A**, **x** for **h**, and **x + 12** for **b** in the formula $A = \frac{1}{2}bh$ yields $270 = \frac{1}{2}(x + 12)(x)$, or $270 = \frac{1}{2}x(x + 12)$. Multiplying both sides of this equation by **2** yields $540 = x(x + 12)$. Applying the distributive property on the right-hand side of this equation yields $540 = x^2 + 12x$. Subtracting **540** from both sides of this equation yields $0 = x^2 + 12x - 540$. In factored form, this equation is equivalent to $(x + 30)(x - 18) = 0$. Applying the zero product property, it follows that $x + 30 = 0$ or $x - 18 = 0$. Subtracting **30** from both sides of the equation $x + 30 = 0$ yields $x = -30$. Adding **18** to both sides of the equation $x - 18 = 0$ yields $x = 18$. Since **x** represents the height of the triangle, it must be positive. Therefore, the height, in centimeters, of the triangle is **18**.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 44665c9b

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 44665c9b

The function f is defined by $f(x) = a(2.2^x + 2.2^b)$, where a and b are integer constants and $0 < a < b$. The functions g and h are equivalent to function f , where k and m are constants. Which of the following equations displays the y -coordinate of the y -intercept of the graph of $y = f(x)$ in the xy -plane as a constant or coefficient?

- I. $g(x) = a(2.2^x + k)$
- II. $h(x) = a(2.2)^x + m$

- A. I only
- B. II only
- C. I and II
- D. Neither I nor II

ID: 44665c9b Answer

Correct Answer: D

Rationale

Choice D is correct. A y -intercept of a graph in the xy -plane is a point where the graph intersects the y -axis, or a point where $x = 0$. Substituting 0 for x in the equation defining function f yields $f(0) = a(2.2^0 + 2.2^b)$, or $f(0) = a(1 + 2.2^b)$. So, the y -coordinate of the y -intercept of the graph is $a(1 + 2.2^b)$, or equivalently, $a + a(2.2)^b$. It's given that function g is equivalent to function f , where $0 < a < b$. It follows that $k = 2.2^b$. Since $a(2.2)^b$ can't be equal to 0 , the coefficient a can't be equal to $a + a(2.2)^b$. Since $0 < a$, the constant k , which is equal to 2.2^b , can't be equal to $a + a(2.2)^b$. Therefore, function g doesn't display the y -coordinate of the y -intercept of the graph of $y = f(x)$ in the xy -plane as a constant or coefficient. It's also given that function h is equivalent to function f , where $0 < a < b$. The equation defining f can be rewritten as $f(x) = a(2.2)^x + a(2.2)^b$. It follows that $m = a(2.2)^b$. Since $a(2.2)^b$ can't be equal to 0 , the coefficient a can't be equal to $a + a(2.2)^b$. Since $0 < a$, the constant m , which is equal to $a(2.2)^b$, can't be equal to $a + a(2.2)^b$. Therefore, function h doesn't display the y -coordinate of the y -intercept of the graph of $y = f(x)$ in the xy -plane as a constant or coefficient. Thus, neither function g nor function h displays the y -coordinate of the y -intercept of the graph of $y = f(x)$ in the xy -plane as a constant or coefficient.

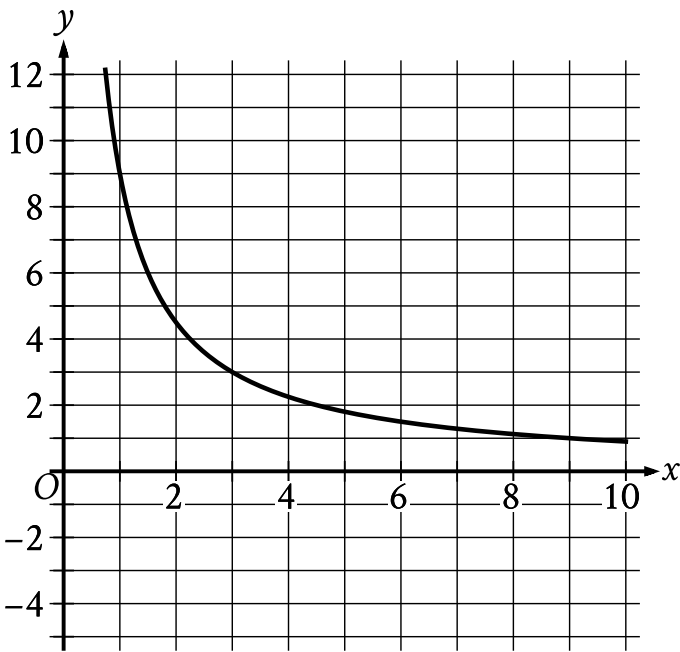
- Choice A is incorrect and may result from conceptual or calculation errors.
- Choice B is incorrect and may result from conceptual or calculation errors.
- Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID bc0f862d

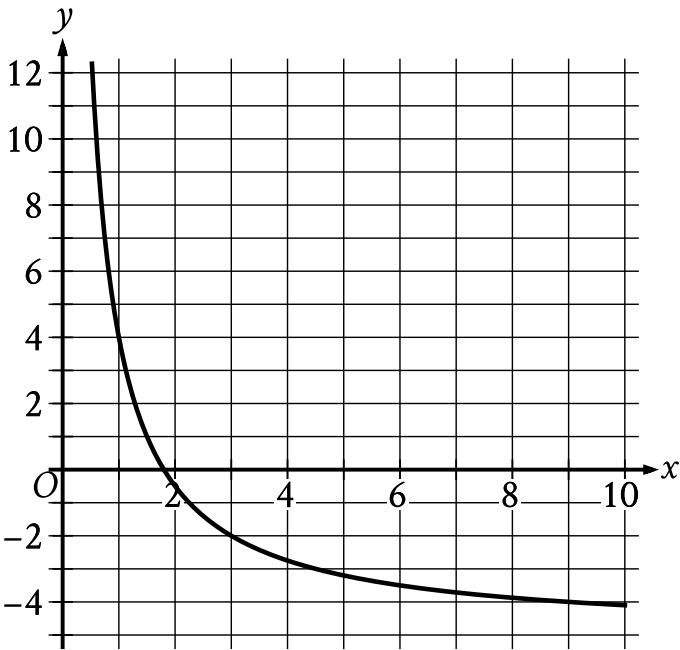
Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: bc0f862d

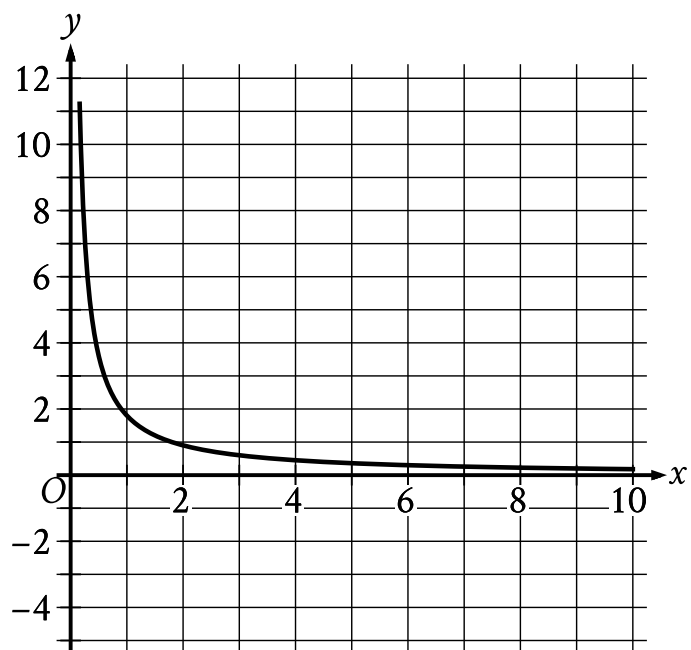


The graph of the rational function f is shown, where $y = f(x)$ and $x \geq 0$. Which of the following is the graph of $y = f(x) + 5$, where $x \geq 0$?

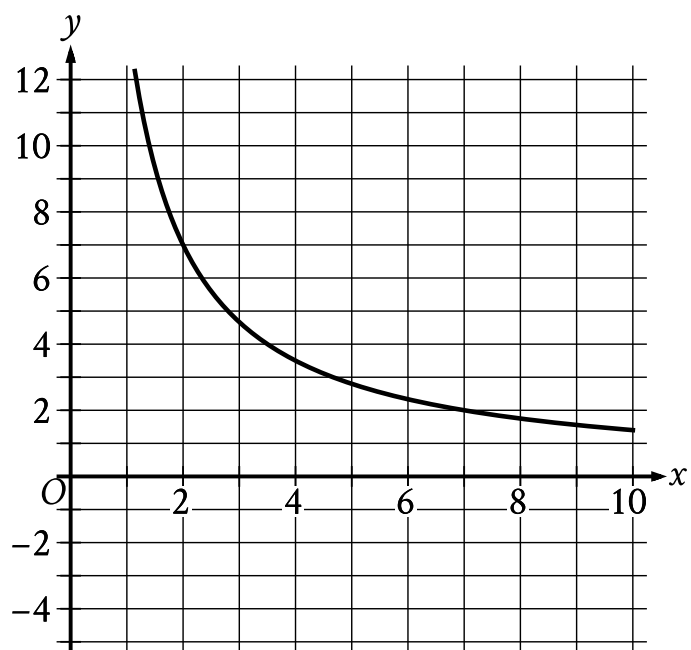
A.



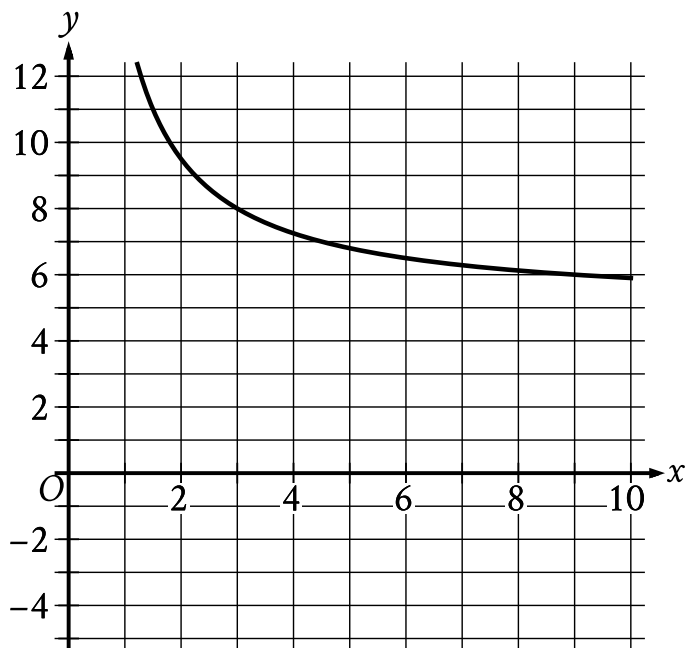
B.



C.



D.



ID: bc0f862d Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the graph of the rational function f is shown, where $y = f(x)$ and $x \geq 0$. The graph shown passes through the point $(3, 3)$. It follows that when the value of x is 3 , the value of $f(x)$ is 3 . When the value of $f(x)$ is 3 , the value of $f(x) + 5$ is $3 + 5$, or 8 . Therefore, the graph of $y = f(x) + 5$ passes through the point $(3, 8)$. Of the given choices, choice D is the only graph that passes through the point $(3, 8)$ and is therefore the graph of $y = f(x) + 5$.

Choice A is incorrect. This is the graph of $y = f(x) - 5$, rather than $y = f(x) + 5$.

Choice B is incorrect. This is the graph of $y = \frac{f(x)}{5}$, rather than $y = f(x) + 5$.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 4471075d

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 4471075d

Function f is a quadratic function where $f(-20) = 0$ and $f(-4) = 0$. The graph of $y = f(x)$ in the xy -plane has a vertex at $(r, -64)$. What is the value of r ?

ID: 4471075d Answer

Correct Answer: -12

Rationale

The correct answer is -12 . It's given that function f is a quadratic function where $f(-20) = 0$ and $f(-4) = 0$. It follows that the graph of $y = f(x)$ in the xy -plane passes through the points $(-20, 0)$ and $(-4, 0)$. When the graph of a quadratic function contains two points $(a, 0)$ and $(b, 0)$, the x -coordinate of the vertex of the graph is the average of a and b . Therefore, the x -coordinate of the vertex of the graph of $y = f(x)$ is $\frac{-20+(-4)}{2}$, or -12 . It's given that the graph of $y = f(x)$ in the xy -plane has a vertex at $(r, -64)$. It follows that the value of r is -12 .

Question Difficulty: Hard

Question ID ac40b851

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: ac40b851

$$p(x) + 57 = x^2$$

The given equation relates the value of x and its corresponding value of $p(x)$ for the function p . What is the minimum value of the function p ?

- A. $-3,249$
- B. -57
- C. 57
- D. $3,249$

ID: ac40b851 Answer

Correct Answer: B

Rationale

Choice B is correct. For a quadratic function defined by an equation of the form $p(x) = a(x - h)^2 + k$, where a , h , and k are constants and $a > 0$, the minimum value of the function is k . Subtracting 57 from both sides of the given equation yields $p(x) = x^2 - 57$. This function is in the form $p(x) = a(x - h)^2 + k$, where $a = 1$, $h = 0$, and $k = -57$. Therefore, the minimum value of the function p is -57 .

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

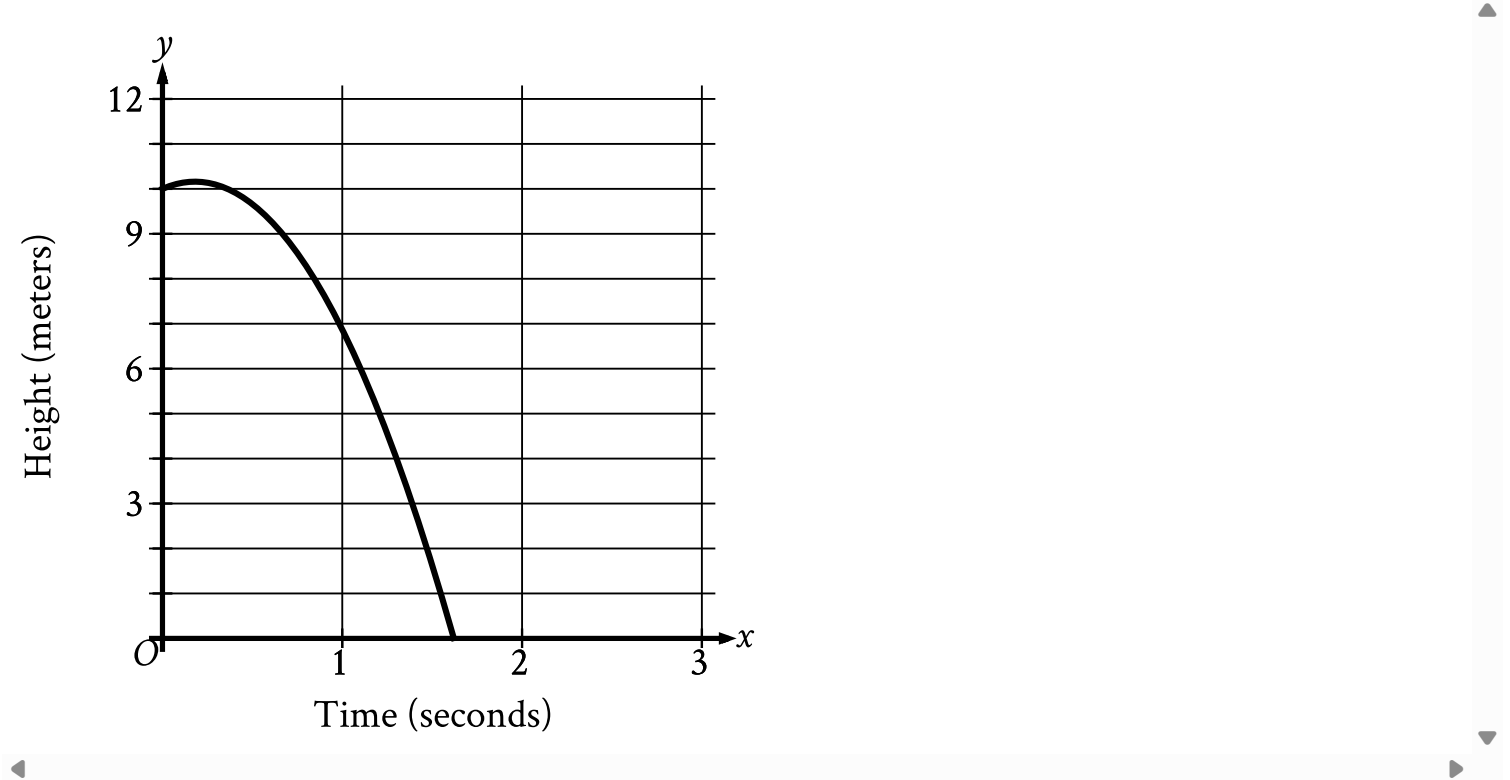
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID bc13249f

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: bc13249f



A competitive diver dives from a platform into the water. The graph shown gives the height above the water y , in meters, of the diver x seconds after diving from the platform. What is the best interpretation of the x-intercept of the graph?

- A. The diver reaches a maximum height above the water at **1.6** seconds.
- B. The diver hits the water at **1.6** seconds.
- C. The diver reaches a maximum height above the water at **0.2** seconds.
- D. The diver hits the water at **0.2** seconds.

ID: bc13249f Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the graph shows the height above the water y , in meters, of a diver x seconds after diving from a platform. The x-intercept of a graph is the point at which the graph intersects the x-axis, or when the value of y is 0. The graph shown intersects the x-axis between $x = 1$ and $x = 2$. In other words, the diver is 0 meters above the water, or hits the water, between 1 and 2 seconds after diving from the platform. Of the given choices, only choice B includes an interpretation where the diver hits the water between 1 and 2 seconds. Therefore, the best interpretation of the x-intercept of the graph is the diver hits the water at **1.6** seconds.

Choice A is incorrect and may result from conceptual errors.

Choice C is incorrect. This is the best interpretation of the maximum value, not the x-intercept, of the graph.

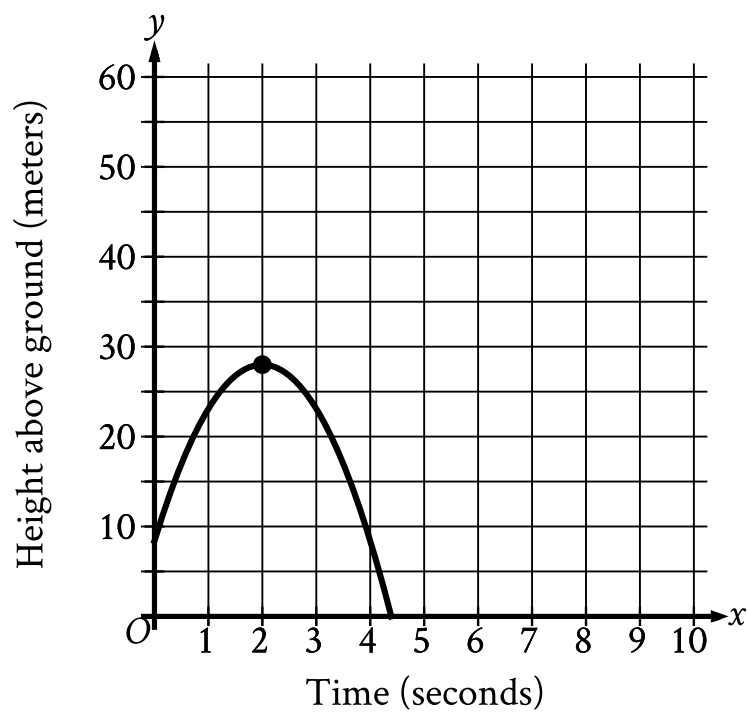
Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 84997c9e

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 84997c9e



An object was launched upward from a platform. The graph shown models the height above ground, y , in meters, of the object x seconds after it was launched. For which of the following intervals of time was the height of the object increasing for the entire interval?

- A. From $x = 0$ to $x = 2$
- B. From $x = 0$ to $x = 4$
- C. From $x = 2$ to $x = 3$
- D. From $x = 3$ to $x = 4$

ID: 84997c9e Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the variable y represents the height, in meters, of the object above the ground. The graph shows that the height of the object was increasing from $x = 0$ to $x = 2$, and decreasing from $x = 2$ to $x = 4$. Therefore, the height of the object was increasing for the entire interval of time from $x = 0$ to $x = 2$.

Choice B is incorrect. The height of the object wasn't increasing for this entire interval of time, as it was decreasing from $x = 2$ to $x = 4$.

Choice C is incorrect. The height of the object was decreasing, not increasing, for this entire interval of time.

Choice D is incorrect. The height of the object was decreasing, not increasing, for this entire interval of time.

Question Difficulty: Hard

Question ID 2e23b002

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 2e23b002

The area of a rectangular banner is **2,661** square inches. The banner's length x , in inches, is **24** inches longer than its width, in inches. Which equation represents this situation?

- A. $0 = x^2 - 24x - 2,661$
- B. $0 = x^2 - 24x + 2,661$
- C. $0 = x^2 + 24x - 2,661$
- D. $0 = x^2 + 24x + 2,661$

ID: 2e23b002 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the banner's length x , in inches, is **24** inches longer than its width, in inches. It follows that the banner's width, in inches, can be represented by the expression $x - 24$. The area of a rectangle is the product of its length and its width. It's given that the area of the banner is **2,661** square inches, so it follows that $2,661 = x(x - 24)$, or $2,661 = x^2 - 24x$. Subtracting **2,661** from each side of this equation yields $0 = x^2 - 24x - 2,661$. Therefore, the equation that represents this situation is $0 = x^2 - 24x - 2,661$.

Choice B is incorrect and may result from representing the width, in inches, of the banner as $24 - x$, rather than $x - 24$.

Choice C is incorrect and may result from representing the width, in inches, of the banner as $x + 24$, rather than $x - 24$.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 55995d34

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 55995d34

A company has a newsletter. In January 2018, there were 1,300 customers subscribed to the newsletter. For the next 24 months after January 2018, the total number of customers subscribed to the newsletter each month was 7% greater than the total number subscribed the previous month. Which equation gives the total number of customers, c , subscribed to the company's newsletter m months after January 2018, where $m \leq 24$?

- A. $c = 1,300^{msup}$
- B. $c = 1,300^{msup}$
- C. $c = 1,300^{msup}$
- D. $c = 1,300^{msup}$

ID: 55995d34 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that in January 2018, there were 1,300 customers subscribed to a company's newsletter and for the next 24 months after January 2018, the total number of customers subscribed to the newsletter each month was 7% greater than the total number subscribed the previous month. It follows that this situation can be represented by the equation $c = a\left(1 + \frac{r}{100}\right)^m$, where c is the total number of customers subscribed to the company's newsletter m months after January 2018, a is the number of customers subscribed to the newsletter in January 2018, and the total number of customers subscribed to the newsletter each month was $r\%$ greater than the total number subscribed the previous month. Substituting 1,300 for a and 7 for r in this equation yields $c = 1,300\left(1 + \frac{7}{100}\right)^m$, or $c = 1,300(1.07)^m$.

Choice A is incorrect. This equation represents a situation where the total number of customers subscribed each month was 93% less, not 7% greater, than the total number subscribed the previous month.

Choice C is incorrect. This equation represents a situation where the total number of customers subscribed each month was 70%, not 7%, greater than the total number subscribed the previous month.

Choice D is incorrect. This equation represents a situation where the total number of customers subscribed each month was 600%, not 7%, greater than the total number subscribed the previous month.

Question Difficulty: Hard

Question ID 86b6d3b3

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 86b6d3b3

The functions f and g are defined by the given equations.

$$f(x) = 3 + \left| -2x - x^2 \right|$$

$$g(w) = \left| \frac{-w}{w-1} \right| - w + 5$$

If $f(-4) = c$, where c is a constant, what is the value of $g(c)$?

ID: 86b6d3b3 Answer

Correct Answer: -4.9, -49/10

Rationale

The correct answer is -4.9 . The value of $f(-4)$ is the value of $f(x)$ when $x = -4$. Substituting -4 for x in the equation $f(x) = 3 + \left| -2x - x^2 \right|$ yields $f(-4) = 3 + \left| -2(-4) - (-4)^2 \right|$, or $f(-4) = 3 + \left| -8 \right|$, which is equivalent to $f(-4) = 3 + 8$, or $f(-4) = 11$. Since it's given that $f(-4) = c$, it follows that $c = 11$ and the value of $g(c)$ is the value of $g(11)$. Substituting 11 for w in the equation $g(w) = \left| \frac{-w}{w-1} \right| - w + 5$ yields $g(11) = \left| \frac{-11}{11-1} \right| - 11 + 5$, or $g(11) = \left| -1.1 \right| - 6$, which is equivalent to $g(11) = 1.1 - 6$, or $g(11) = -4.9$. Note that -4.9 and -49/10 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 3d0755fb

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 3d0755fb

When the quadratic function f is graphed in the xy -plane, where $y = f(x)$, its vertex is $(-3, 6)$. One of the x -intercepts of this graph is $(-\frac{17}{4}, 0)$. What is the other x -intercept of the graph?

- A. $(-\frac{29}{4}, 0)$
- B. $(-\frac{7}{4}, 0)$
- C. $(\frac{5}{4}, 0)$
- D. $(\frac{17}{4}, 0)$

ID: 3d0755fb Answer

Correct Answer: B

Rationale

Choice B is correct. Since the line of symmetry for the graph of a quadratic function contains the vertex of the graph, the x -coordinate of the vertex of the graph of $y = f(x)$ is the x -coordinate of the midpoint of its two x -intercepts. The midpoint of two points with x -coordinates x_1 and x_2 has x -coordinate x_m , where $x_m = \frac{x_1+x_2}{2}$. It's given that the vertex is $(-3, 6)$ and one of the x -intercepts is $(-\frac{17}{4}, 0)$. Substituting -3 for x_m and $-\frac{17}{4}$ for x_1 in the equation $x_m = \frac{x_1+x_2}{2}$ yields $-3 = \frac{-\frac{17}{4}+x_2}{2}$. Multiplying each side of this equation by 2 yields $-6 = -\frac{17}{4} + x_2$. Adding $\frac{17}{4}$ to each side of this equation yields $-\frac{7}{4} = x_2$. Therefore, the other x -intercept is $(-\frac{7}{4}, 0)$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 5e2487bb

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: 5e2487bb

For the exponential function f , the value of $f(0)$ is c , where c is a constant. Of the following equations that define the function f , which equation shows the value of c as the coefficient or the base?

- A. $f(x) = 22(1.5)^{x+1}$
- B. $f(x) = 33(1.5)^x$
- C. $f(x) = 49.5(1.5)^{x-1}$
- D. $f(x) = 74.25(1.5)^{x-2}$

ID: 5e2487bb Answer

Correct Answer: B

Rationale

Choice B is correct. Each of the given choices is an equation of the form $f(x) = a(b)^{x-k}$, where a , b , and k are constants. For an equation of this form, the coefficient, a , is equal to the value of the function when the exponent is equal to 0, or when $x = k$. It follows that in the equation $f(x) = 33(1.5)^x$, the coefficient, 33, is equal to the value of $f(0)$. Substituting 0 for x in this equation yields $f(0) = 33(1.5)^0$, which is equivalent to $f(0) = 33(1)$, or $f(0) = 33$. Thus, the value of c is 33 and the equation $f(x) = 33(1.5)^x$ shows the value of c as the coefficient.

Choice A is incorrect. This equation shows the value of $f(-1)$, not $f(0)$, as the coefficient.

Choice C is incorrect. This equation shows the value of $f(1)$, not $f(0)$, as the coefficient.

Choice D is incorrect. This equation shows the value of $f(2)$, not $f(0)$, as the coefficient.

Question Difficulty: Hard

Question ID cbd9bf3f

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: cbd9bf3f

The product of a positive number x and the number that is 8 more than x is 180. What is the value of x ?

- A. 5
- B. 10
- C. 18
- D. 36

ID: cbd9bf3f Answer

Correct Answer: B

Rationale

Choice B is correct. The number that's 8 more than x can be represented by the expression $x + 8$. It's given that the product of x and $x + 8$ is 180, so it follows that $(x)(x + 8) = 180$, or $x^2 + 8x = 180$. Subtracting 180 from each side of this equation yields $x^2 + 8x - 180 = 0$. Factoring the left-hand side of this equation yields $(x - 10)(x + 18) = 0$. Applying the zero product property to this equation yields two solutions: $x = 10$ and $x = -18$. Since x is a positive number, the value of x is 10.

Choice A is incorrect. If $x = 5$, the product of x and the number that's 8 more than x would be $(5)(13)$, or 65, not 180.

Choice C is incorrect. This is the value of the number that's 8 more than x , not the value of x .

Choice D is incorrect. If $x = 36$, the product of x and the number that's 8 more than x would be $(36)(44)$, or 1,584, not 180.

Question Difficulty: Hard

Question ID bb682312

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: bb682312

The function $f(x) = \frac{1}{9}(x - 7)^2 + 3$ gives a metal ball's height above the ground $f(x)$, in inches, x seconds after it started moving on a track, where $0 \leq x \leq 10$. Which of the following is the best interpretation of the vertex of the graph of $y = f(x)$ in the xy -plane?

- A. The metal ball's minimum height was 3 inches above the ground.
- B. The metal ball's minimum height was 7 inches above the ground.
- C. The metal ball's height was 3 inches above the ground when it started moving.
- D. The metal ball's height was 7 inches above the ground when it started moving.

ID: bb682312 Answer

Correct Answer: A

Rationale

Choice A is correct. The graph of a quadratic equation in the form $y = a(x - h)^2 + k$, where a , h , and k are positive constants, is a parabola that opens upward with vertex (h, k) . The given function $f(x) = \frac{1}{9}(x - 7)^2 + 3$ is in the form $y = a(x - h)^2 + k$, where $y = f(x)$, $a = \frac{1}{9}$, $h = 7$, and $k = 3$. Therefore, the graph of $y = f(x)$ is a parabola that opens upward with vertex $(7, 3)$. Since the parabola opens upward, the vertex is the lowest point on the graph. It follows that the y -coordinate of the vertex of the graph of $y = f(x)$ is the minimum value of $f(x)$. Therefore, the minimum value of $f(x)$ is 3. It's given that $f(x) = \frac{1}{9}(x - 7)^2 + 3$ represents the metal ball's height above the ground, in inches, x seconds after it started moving on a track. Therefore, the best interpretation of the vertex of the graph of $y = f(x)$ is that the metal ball's minimum height was 3 inches above the ground.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID c41ef15e

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear functions	Hard

ID: c41ef15e

The functions g and h are defined by the given equations, where $x \geq 0$. Which of the following equations displays, as a constant or coefficient, the minimum value of the function it defines, where $x \geq 0$?

I. $g(x) = 18(1.16)(1.4)^{x+2}$

II. $h(x) = 18(1.4)^{x+4}$

- A. I only
- B. II only
- C. I and II
- D. Neither I nor II

ID: c41ef15e Answer

Correct Answer: D

Rationale

Choice D is correct. A function defined by an equation in the form $f(x) = a(b)^{x+h}$, where a , b , and h are positive constants and $x \geq 0$, has a minimum value of $f(0)$. It's given that function g is defined by $g(x) = 18(1.16)(1.4)^{x+2}$, which is equivalent to $g(x) = 20.88(1.4)^{x+2}$. Substituting 0 for x in this equation yields $g(0) = 20.88(1.4)^{0+2}$, or $g(0) = 40.9248$. Therefore, the minimum value of $g(x)$ is 40.9248, so $g(x) = 18(1.16)(1.4)^{x+2}$ doesn't display its minimum value as a constant or coefficient. It's also given that function h is defined by $h(x) = 18(1.4)^{x+4}$. Substituting 0 for x in this equation yields $h(0) = 18(1.4)^{0+4}$, or $h(0) = 69.1488$. Therefore, the minimum value of $h(x)$ is 69.1488, so $h(x) = 18(1.4)^{x+4}$ doesn't display its minimum value as a constant or coefficient. Therefore, neither I nor II displays, as a constant or coefficient, the minimum value of the function it defines, where $x \geq 0$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard